Electromagnetic Waves

plane wave: linearly polarized

Most light we encounter is not polarized
## Electromagnetic Spectrum

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<th>Radiation</th>
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<td>Navigation</td>
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</table>
Surfaces of Constant Phase Define “Wave Fronts”

Wave front:
locus of all adjacent points that are at same point in cycle of oscillation
“Rays” are lines perpendicular to wave fronts

(a)

(b)
Law of Reflection

\[ \theta = \theta' \]
Law of Reflection
Speed of Light

\[ v = c \]  
\[ v_m = \frac{c}{n} \]

in vacuum
in a material
index of refraction
Refraction: Snell’s Law

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]
Refraction is a consequence of change in speed

During time $dt$: ray 1 travels $v_1dt$
ray 2 travels $v_2dt$
$v_2=(c/n)<v_1$

$n_2>n_1$
Total Internal Reflection

\[ \sin \theta_c = \frac{n_2}{n_1} \]
Light ray is “trapped” by internal reflections if $\alpha$, $\beta$, and $\gamma$ exceed the critical angle.
Mirages

Cooler air

Hot air
(a) Paths of rays entering upper half of raindrop (for clarity, rays entering lower half are not shown)

\[ \Delta = \text{maximum angle of light from raindrop} \]

\[ \Delta = 50.1^\circ \text{ (red)} \text{ to } 53.2^\circ \text{ (violet)} \]

(b) Primary rainbow

\[ \Delta = 40.8^\circ \text{ (violet)} \text{ to } 42.5^\circ \text{ (red)} \]

(c) Secondary rainbow
\[ \vec{E}(x, t) = \hat{\mathbf{j}} E_{\text{max}} \cos(kx - \omega t) \]

\[ \vec{E}(x, t) = \hat{\mathbf{k}} E_{\text{max}} \cos(kx - \omega t) \]
Incident natural light

Polarizing axis

Polaroid filter

Light with horizontal polarization almost completely absorbed

Light with vertical polarization partially absorbed

Linearly polarized transmitted light
Incident natural light

Polarizing axis

Transmitted light, linearly polarized parallel to polarizing axis

Photocell
Malus’ Law
(linearly pol. light)

\[ I \propto \omega^2 A^2 \quad (I \propto \omega^2 E_{\text{max}}^2) \]

\[ I = I_{\text{max}} \cos^2 \phi \]
Polarization by Reflection

Complete polarization for $\theta_p + \theta_b = 90$:

\[ n_a \sin \theta_p = n_b \sin \theta_b \]

\[ n_a \sin \theta_p = n_b \sin(90 - \theta_p) = n_b \cos \theta_p \]

\[ \Rightarrow \tan \theta_p = \frac{n_b}{n_a} \quad \text{(Brewster’s law)} \]
Linear Polarization: Simple LCD Displays (e.g. watches)

Display pixel appears gray
molecular axis rotates between plates
liquid crystal molecules

Display pixel appears black
applied voltage aligns molecule axes \( \perp \) to plates

http://www.polarization.com/beetle/beetle.html
Circular Polarization: glare reduction at the ATM

http://www.polarization.com/beetle/beetle.html
Refraction at a spherical surface

Key assumptions:

1) angles $\alpha, \beta, \phi$ small $\Rightarrow \sin \alpha \approx \tan \alpha \approx \alpha$, etc. for $\beta, \phi$

2) Take $\delta \approx 0$ (follows from $\phi$ being small)

\[
\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}
\]
Sign conventions

\[ \frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \]

R>0 when C is on outgoing side of interface
R<0 when C is on incoming side of interface

s>0 when object is on incoming side of interface
s<0 when object is on outgoing side of interface
Thin Lenses

\[
\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1}
\]

\[
\frac{n_b}{s_2} + \frac{n_c}{s_2'} = \frac{n_c - n_b}{R_2}
\]
Typically, \( n_a = n_c = 1 \) (air) and \( n_b = n \):

\[
\frac{1}{s_1} + \frac{n}{s'_1} = \frac{n-1}{R_1} \quad (1)
\]

\[
\frac{n}{s_2} + \frac{1}{s'_2} = \frac{1-n}{R_2} \quad (2)
\]

Image from first refraction serves as object for 2nd:

\[ s_2 = -s'_1 \]

Combine (1) and (2) to get expression relating \( s_1 \) and \( s'_2 \) (drop subscripts)

\[
\Rightarrow \frac{1}{s} + \frac{1}{s'} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f}
\]

\[
f > 0 \quad \rightarrow \quad \text{converging lens}
\]

\[
f < 0 \quad \rightarrow \quad \text{diverging lens}
\]
Converging lens: $R_1 > 0$ and $R_2 < 0$

\[
\frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]

\[\Rightarrow f > 0\]