Gravity Without Dark Matter

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Contents

1. Introduction

- Can gravity theory explain galaxy dynamics and cosmology without dominant dark matter?
- We need a gravity theory that generalizes GR. Such a generalization is nonsymmetric gravitational theory (NGT).
- A simpler theory is metric-skew-tensor gravity (MSTG).
- There is also the problem of the nature of Dark Energy that explains the accelerating Universe.
- The gravity theory must be stable and contain GR in a consistent way. It must yield agreement with solar system tests and the binary pulsar PSR 1913+16.
- The cosmology without dominant dark matter must agree with the latest WMAP and CMB data.

2. Gravity theory

- A popular phenomenological model that replaces dark matter is MOND (Milgrom, Beckenstein, Sanders-McGough). This phenomenology fits the flat rotation curves of galaxies, but has no acceptable relativistic gravity theory (see, however, Beckenstein 2004).

- The MSTG theory can explain flat rotation curves of galaxies and cosmology without dominant dark matter.

- We need to incorporate quantum gravity in a renormalization group (RG) flow framework with effective running coupling "constants" and an effective action.

- We aim for a non-perturbative renormalizable gravity theory which is ultraviolet (UV) asymptotically safe.

- There should exist infrared (IR) fixed points that allow the gravitational constant G and the skew field coupling constant g(x) to run with momentum k, distance x and time t.

- The G(x) and g(x) run in the IR to galactic and cosmological distance scales (see, Reuter and Weyer 2004).

3. The action and field equations

The action is given by

$$S = S_G + S_F + S_{FM} + S_M$$

$$S_g = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R - 2\Lambda)$$

$$S_F = \int d^4x\sqrt{-g}\left(\frac{1}{12}F_{\mu\nu\rho}F^{\mu\nu\rho} - \frac{1}{4}\mu^2 A_{\mu\nu}A^{\mu\nu}\right)$$

$$F_{\mu\nu\lambda} = \partial_\mu A_{\nu\lambda} + \partial_\nu A_{\lambda\mu} + \partial_\lambda A_{\mu\nu}$$

$$G = G_0 Z.$$

The matter actions satisfy

$$\frac{1}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\mu\nu}} = -\frac{1}{2}T_{M\mu\nu}, \quad \frac{1}{\sqrt{-g}}\frac{\delta S_F}{\delta g^{\mu\nu}} = -\frac{1}{2}T_{F\mu\nu}$$

$$\frac{\delta S_{FM}}{\delta A^{\mu\nu}} = -J_{\mu\nu}$$

The field equations are

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\nabla^{\sigma} F_{\mu\nu\sigma} + \mu^2 A_{\mu\nu} = \frac{1}{\sqrt{-g}} J_{\mu\nu}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \quad T_{\mu\nu} = T_{M\mu\nu} + T_{F\mu\nu}$$

A possible skew field action is

$$S_{FM} = \int d^4x F_{\lambda\mu\nu} J^{*\lambda\mu\nu} = -3 \int d^4x \epsilon^{\alpha\beta\mu\nu} A_{\alpha\beta} \partial_\mu J_\nu$$

The equations of motion of a test particle are

$$\frac{du^\mu}{d\tau} + \left\{ {\mu \atop \alpha\beta} \right\} u^\alpha u^\beta = g^{\mu\alpha} f_{\alpha\nu} u^\nu$$

$$f_{\alpha\mu} = \lambda \partial_{[\alpha} \left(\frac{\epsilon^{\eta\sigma\nu\lambda}}{\sqrt{-g}} H_{\sigma\nu\lambda} g_{\mu]\eta} \right)$$

$$H_{\mu\nu\lambda} = \frac{1}{3}(\partial_\lambda A_{\mu\nu} + \partial_\mu A_{\nu\lambda} + \partial_\nu A_{\lambda\mu})$$

We expand around Minkowski flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$$

The skew field satisfies the linearized equations of motion in empty space

$$\partial^\sigma F_{\mu\nu\sigma} + \mu^2 A_{\mu\nu} = 0.$$

We choose for a static spherically symmetric field only the "electric" component:

$$A_{\theta\phi}(r) = f(r)\sin\theta.$$

$$f(r) = \frac{1}{3} sG^2M^2 \exp(-\mu r)(1+\mu r).$$

We shall postulate scale dependent effective actions $\Gamma_k[g_{\mu\nu}]$ and $\Gamma_k[A_{\mu\nu}]$ corresponding to "coarse-grained" free energy functionals, which define an effective action field theory valid at a mass scale k or length scale l_s =1/k. The Γ_k are determined by functional differential equations for the RG flow, including the dimensionless coupling constants $\bar{G}(k) = k^2 G(k), \gamma_c(k)$ the cosmological constant $\lambda(k) = \Lambda(k)/k^2$ and the mass $\bar{\mu} = \mu(k)/k$.

The flow equations are

$$k\frac{d}{dk}\bar{G}(k) = \beta_{\bar{G}}(\bar{G}, \gamma_c, \lambda, \bar{\mu}), \quad k\frac{d}{dk}\gamma_c(k) = \beta_{\gamma_c}(\bar{G}, \gamma_c, \lambda, \bar{\mu}),$$

$$k\frac{d}{dk}\lambda(k) = \beta_{\lambda}(\bar{G}, \gamma_c, \lambda, \bar{\mu}), \quad k\frac{d}{dk}\mu(k) = \beta_{\bar{\mu}}(\bar{G}, \gamma_c, \lambda, \bar{\mu}).$$

The flow equations are described by the running "constants"

$$G(k) = \bar{G}(k)/k^2, \quad \gamma_c(k), \quad \Lambda(k) = \lambda k^2, \quad \mu(k) = \bar{\mu}(k)k$$

We have to solve the effective equations of motion

$$\frac{\delta\Gamma[g_{\mu\nu}]}{\delta g_{\mu\nu}} = 0, \qquad \frac{\delta\Gamma[A_{\mu\nu}]}{\delta A_{\mu\nu}} = 0.$$

The RG flow equations are dominated by two fixed points G_* and $\bar{\gamma}_{c*}$. The high-energy, short distance behavior of the QG is governed by the non-Gaussian fixed points. If $G(k)$ and $\gamma_c(k)$ are asymptotically safe and vanish as k tends to infinity, then quantum MSTG is a non-perturbatively renormalizable theory.

When we convert from k space to x space, then G =G(x) will run with distance scale. G(x) will increase in the infrared (IR) limit for x of galactic and cosmological distance scale. Thus, the strong IR renormalization effects will have astrophysical and cosmological consequences.

5. Modified acceleration law and flat galaxy rotation curves

The radial acceleration experienced by a test particle for weak static, spherically symmetric fields is

$$a(r) = -\frac{G_\infty M}{r^2} + \sigma \frac{\exp(-r/r_0)}{r^2}\left(1 + \frac{r}{r_0}\right) \qquad G_\infty = G_0\left(1 + \sqrt{\frac{M_0}{M}}\right)$$

$$\sigma = \frac{\lambda s G_0^2 M^2}{3c^2 r_0^2} \qquad s = gM^a$$

We choose a=-3/2 and set $\lambda g G_0/3c^2 r_0^2 = \sqrt{M_0}$. We obtain the acceleration law

$$a(r) = -\frac{G_0 M}{r^2}\left\{1 + \sqrt{\frac{M_0}{M}}\left[1 - \exp(-r/r_0)\left(1 + \frac{r}{r_0}\right)\right]\right\}$$

We can express a(r) for a point mass source as

$$a(r) = -\frac{G(r)M}{r^2}$$

$$G(r) = G_0\left\{1 + \sqrt{\frac{M_0}{M}}\left[1 - \exp(-r/r_0)\left(1 + \frac{r}{r_0}\right)\right]\right\}$$

The rotational velocity of a star is

$$v = \sqrt{\frac{G_0 M}{r}}\left\{1 + \sqrt{\frac{M_0}{M}}\left[1 - \exp(-r/r_0)\left(1 + \frac{r}{r_0}\right)\right]\right\}^{1/2}$$

Consider the constant acceleration

$$a_0 = \frac{G_0 M_0}{r_0^2}$$

$$a_0 = cH_0$$

$$a_0 = cH_0 \sim (\sqrt{\Lambda/3})c^2$$

$$a_0 = 6.90 \times 10^{-8}\,\mathrm{cm\,s^{-2}}$$

A good fit to galaxies is obtained with

$$M_0 = 9.60 \times 10^{11} M_\odot, \quad r_0 = 13.92\,\mathrm{kpc} = 4.30 \times 10^{22}\,\mathrm{cm}$$

r_0 is fixed by the value of a_0 above.

A model of a galaxy with a core density of visible matter is

$$a(r) = -\frac{G_0 \mathcal{M}(r)}{r^2}\left\{1+\sqrt{\frac{M_0}{M}}\left[1-\exp(-r/r_0)\left(1+\frac{r}{r_0}\right)\right]\right\}$$ $$\mathcal{M}(r) = 4\pi\int_0^r dr' r'^2 \rho_c(r')$$ $$\mathcal{M}(r) = M\left(\frac{r}{r_c+r}\right)^2$$

In Milgrom's phenomenological model

$$v^4 = G_0 M (a_0)_{\text{Milgrom}}$$

V is predicted to extend as a constant to infinity. In MSTG:

$$v \sim \sqrt{G_\infty M/r} \text{ as } r \to \infty$$

The predicted v from MSTG fits low surface brightness galaxies, high surface brightness galaxies, elliptical galaxies, dwarf galaxies and globular clusters. The fits for the estimated M (M/L) are as good as Milgrom's MOND (Brownstein and JWM).

A determination of the velocities of satellites orbiting isolated galaxies was performed by Prada et al. 2003, using SDSS to probe the halo mass distributions of galaxies. After removing satellite “interlopers”, they found that the velocities of satellites decline with distance from the primary.

This result agrees with N-body simulation calculations in cosmology, which show that for large distances from the primary galaxy (r ~ 350 kpc), rho ~ 1/r^3 for “dark matter” densities, and not like rho ~ 1/r^2 as predicted by MOND. We predict from MSTG that v^2 ~ G_{inf}M/r, which is consistent with the Prada et al. results. These observations rule out MOND.

The orbital equation of motion is

$$\frac{d^2u}{d\phi^2} + u = \frac{GM}{c^2J^2} - \frac{K}{J^2}\exp(-r/r_0)\left[1+\left(\frac{r}{r_0}\right)\right] + \frac{3GM}{c^2}u^2$$

$$K = \lambda s G^2 M^2 / 3c^4 r_0^2.$$

For r << r_0 the orbit equation becomes

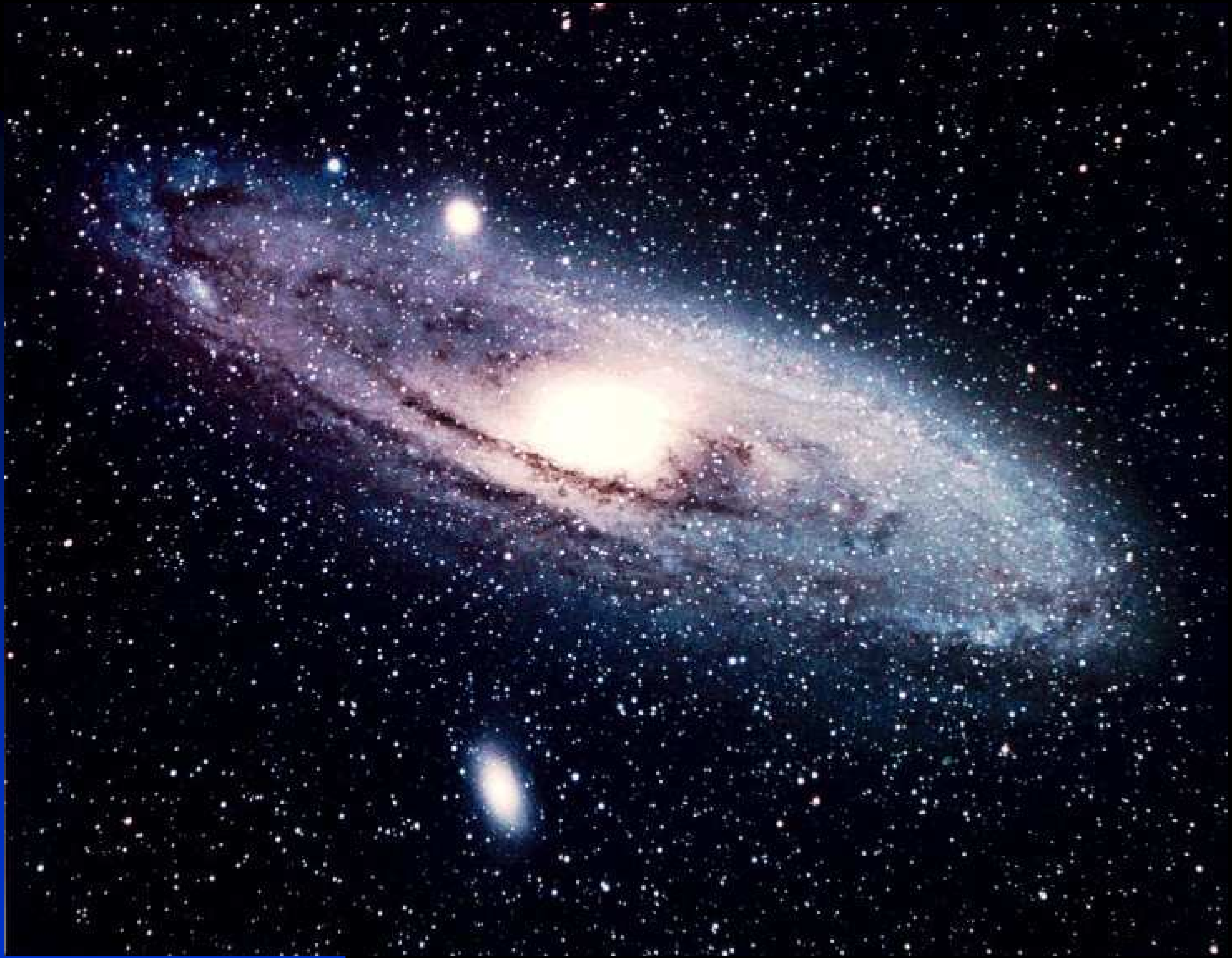
$$\frac{d^2u}{d\phi^2} + u = N + 3\frac{GM}{c^2}u^2$$

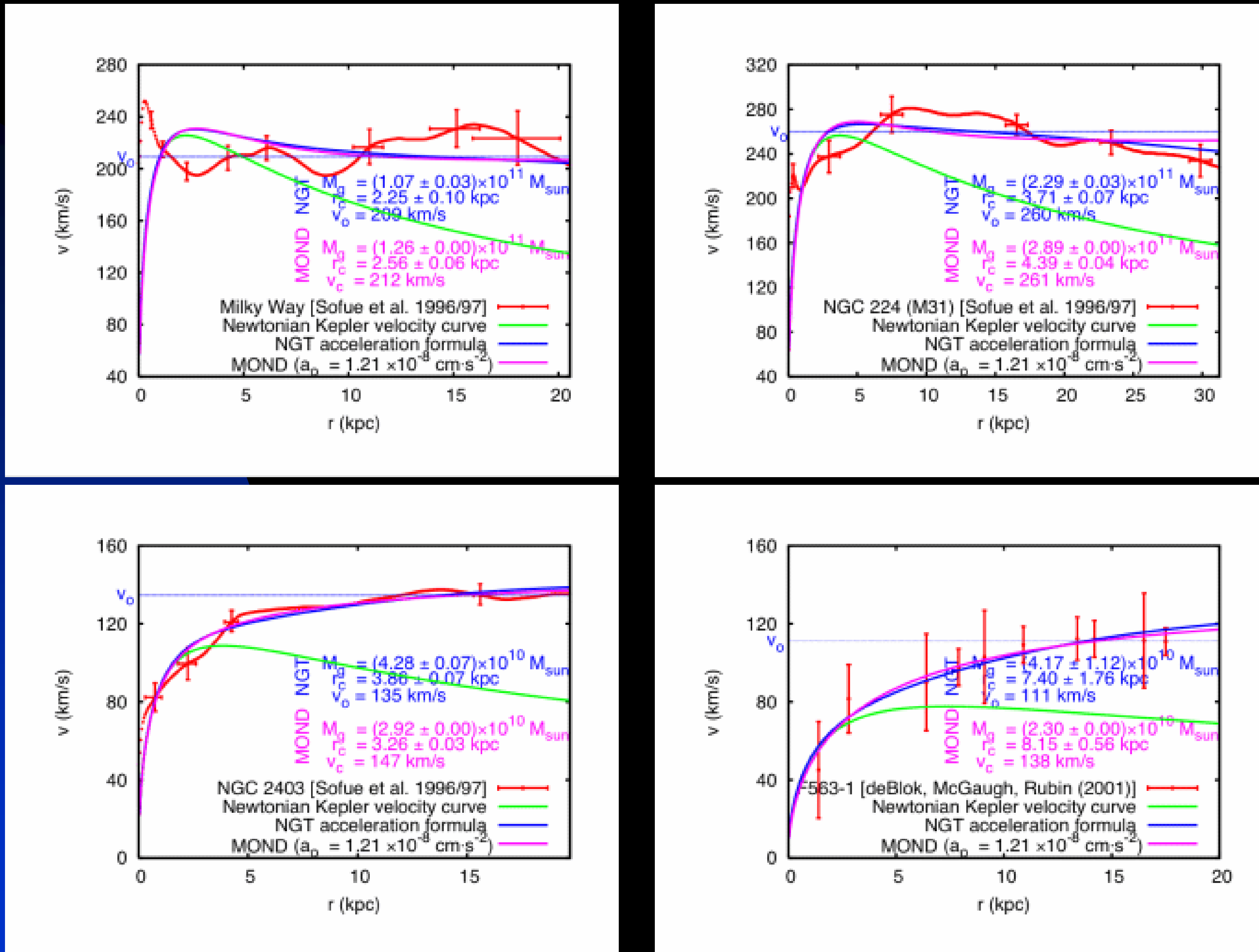
$$N = \frac{GM}{c^2 J_N^2} - \frac{K}{J_N^2}$$

We can solve by perturbation theory for the perihelion advance of Mercury:

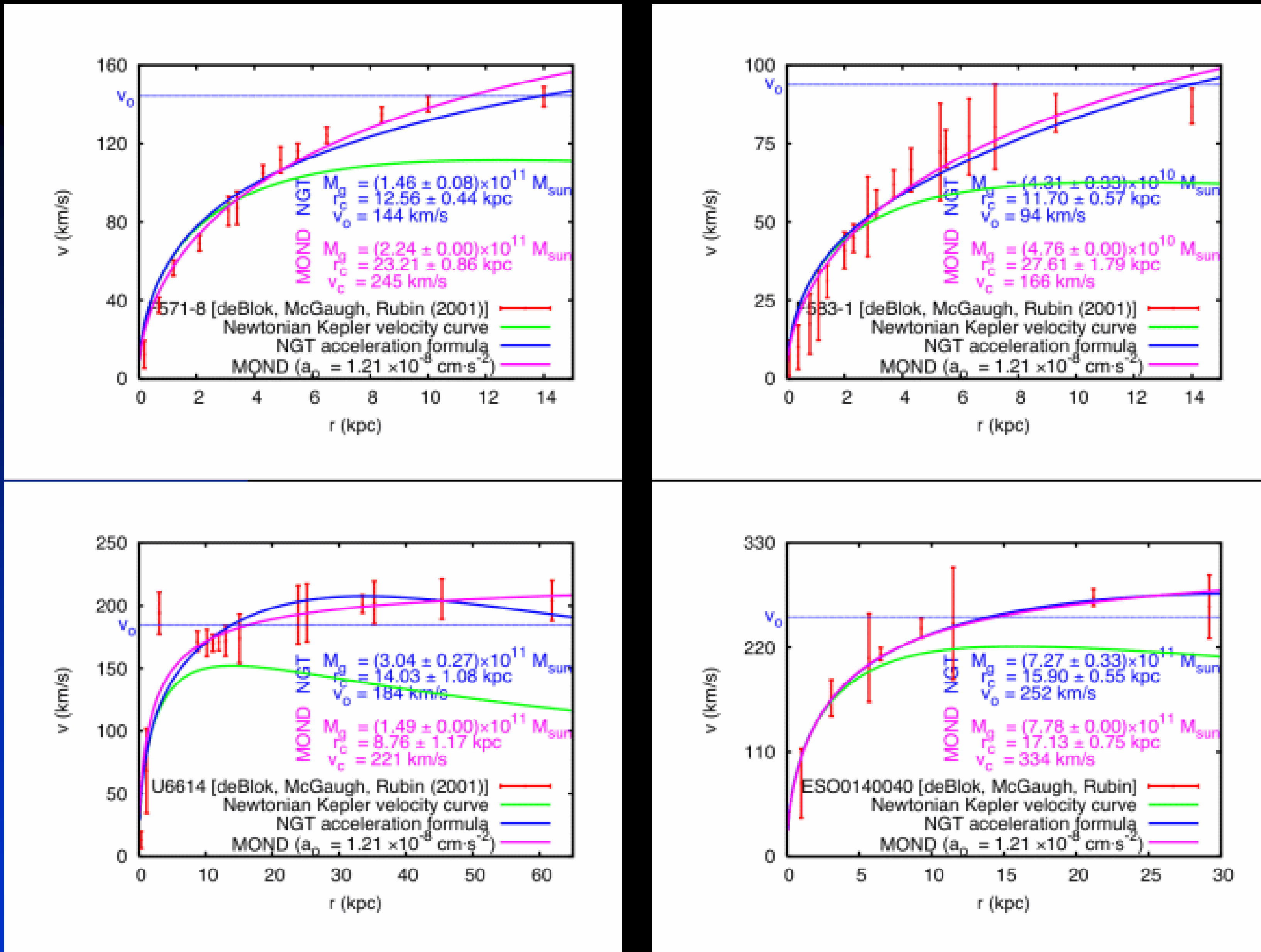
$$\Delta\omega = \frac{6\pi}{c^2 L}(GM_\odot - c^2 K_\odot).$$

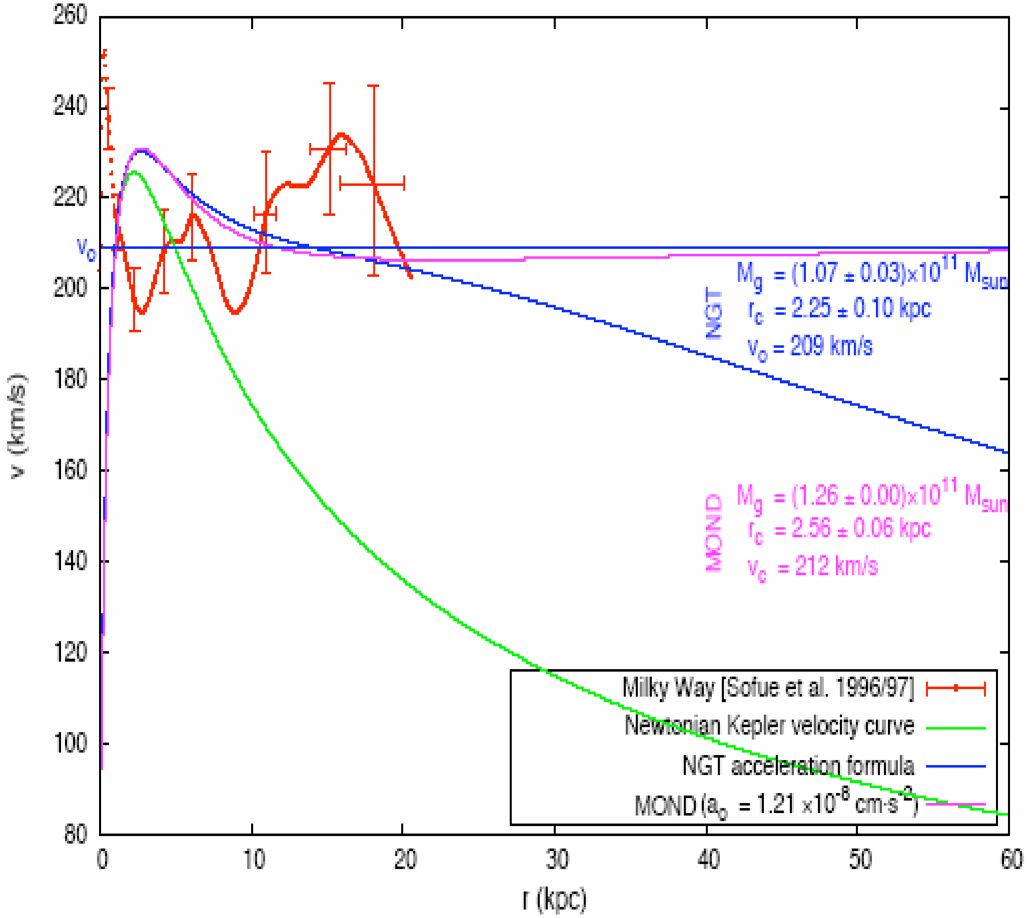
We now use the renormalization of the effective G from the running of G(r) and for r << 14 kpc, we have G ~ G_0 yielding agreement with GR. The bending of light in the solar system and the binary pulsar PSR 1913+16 observations also agree with GR. Bounds from “fifth force” experiments only apply to the solar system.





v (km/s)
r (kpc)
NGT
M_g = (1.07 ± 0.03)×10^11 M_sun
r_c = 2.25 ± 0.10 kpc
v_o = 209 km/s
MOND
M_g = (1.26 ± 0.00)×10^11 M_sun
r_c = 2.56 ± 0.06 kpc
v_c = 212 km/s
Milky Way [Sofue et al. 1996/97]
Newtonian Kepler velocity curve
NGT acceleration formula
MOND (a_0 = 1.21 ×10^-8 cm·s^-2)
NGT
M_g = (2.29 ± 0.03)×10^11 M_sun
r_c = 3.71 ± 0.07 kpc
v_o = 260 km/s
MOND
M_g = (2.89 ± 0.00)×10^11 M_sun
r_c = 4.39 ± 0.04 kpc
v_c = 261 km/s
NGC 224 (M31) [Sofue et al. 1996/97]
NGT
M_g = (4.28 ± 0.07)×10^10 M_sun
r_c = 3.86 ± 0.07 kpc
v_o = 135 km/s
MOND
M_g = (2.92 ± 0.00)×10^10 M_sun
r_c = 3.26 ± 0.03 kpc
v_c = 147 km/s
NGC 2403 [Sofue et al. 1996/97]
NGT
M_g = (4.17 ± 1.12)×10^10 M_sun
r_c = 7.40 ± 1.76 kpc
v_o = 111 km/s
MOND
M_g = (2.30 ± 0.00)×10^10 M_sun
r_c = 8.15 ± 0.56 kpc
v_c = 138 km/s
F563-1 [deBlok, McGaugh, Rubin (2001)]


v (km/s)
r (kpc)
v0
NGT
Mg = (1.46 ± 0.08)×10^11 Msun
rc = 12.56 ± 0.44 kpc
v0 = 144 km/s
MOND
Mg = (2.24 ± 0.00)×10^11 Msun
rc = 23.21 ± 0.86 kpc
vc = 245 km/s
571-8 [deBlok, McGaugh, Rubin (2001)]
Newtonian Kepler velocity curve
NGT acceleration formula
MOND (a0 = 1.21 ×10^-8 cm·s^-2)
NGT
Mg = (4.31 ± 0.33)×10^10 Msun
rc = 11.70 ± 0.57 kpc
v0 = 94 km/s
MOND
Mg = (4.76 ± 0.00)×10^10 Msun
rc = 27.61 ± 1.79 kpc
vc = 166 km/s
583-1 [deBlok, McGaugh, Rubin (2001)]
Newtonian Kepler velocity curve
NGT acceleration formula
MOND (a0 = 1.21 ×10^-8 cm·s^-2)
NGT
Mg = (3.04 ± 0.27)×10^11 Msun
rc = 14.03 ± 1.08 kpc
v0 = 184 km/s
MOND
Mg = (1.49 ± 0.00)×10^11 Msun
rc = 8.76 ± 1.17 kpc
vc = 221 km/s
U6614 [deBlok, McGaugh, Rubin (2001)]
Newtonian Kepler velocity curve
NGT acceleration formula
MOND (a0 = 1.21 ×10^-8 cm·s^-2)
NGT
Mg = (7.27 ± 0.33)×10^11 Msun
rc = 15.90 ± 0.55 kpc
v0 = 252 km/s
MOND
Mg = (7.78 ± 0.00)×10^11 Msun
rc = 17.13 ± 0.75 kpc
vc = 334 km/s
ESO0140040 [deBlok, McGaugh, Rubin]
Newtonian Kepler velocity curve
NGT acceleration formula
MOND (a0 = 1.21 ×10^-8 cm·s^-2)



v (km/s)
r (kpc)
v_0
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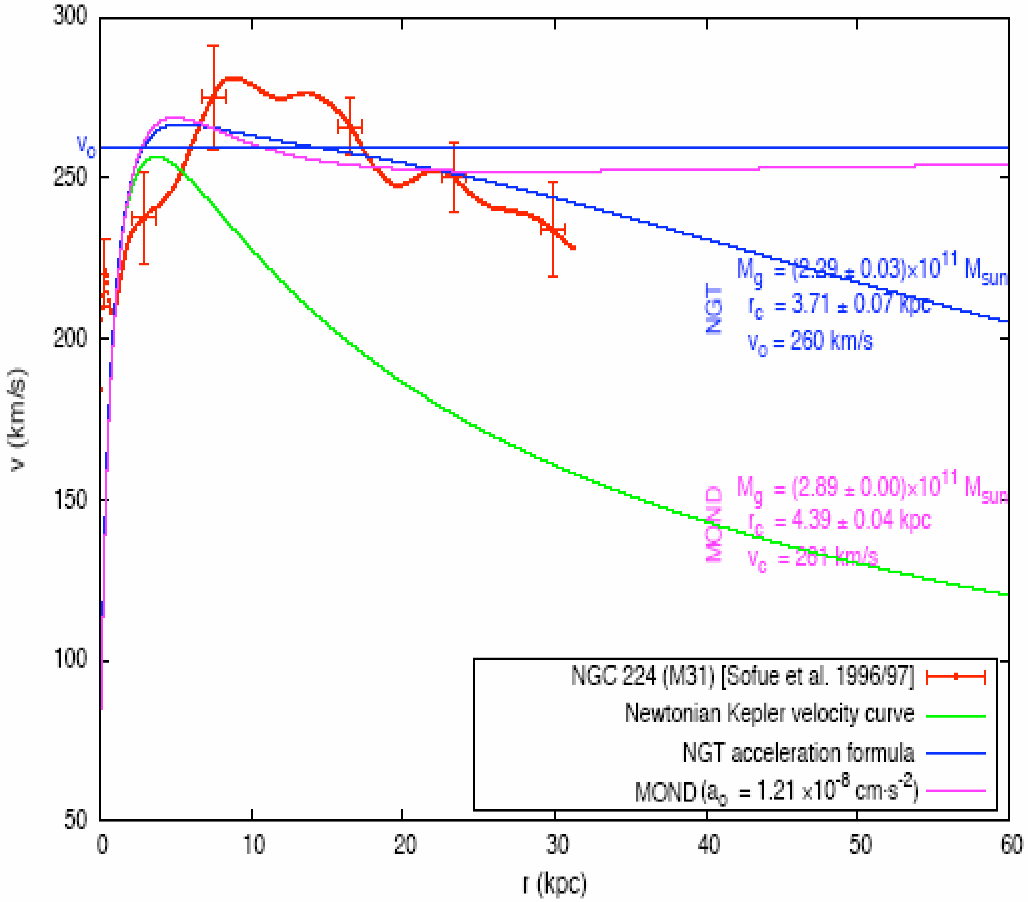

300
250
200
150
100
50
v (km/s)
0
10
20
30
40
50
60
r (kpc)
v_0
NGT
M_g = (2.29 ± 0.03)x10^{11} M_{sun}
r_c = 3.71 ± 0.07 kpc
v_0 = 260 km/s
MOND
M_g = (2.89 ± 0.00)x10^{11} M_{sun}
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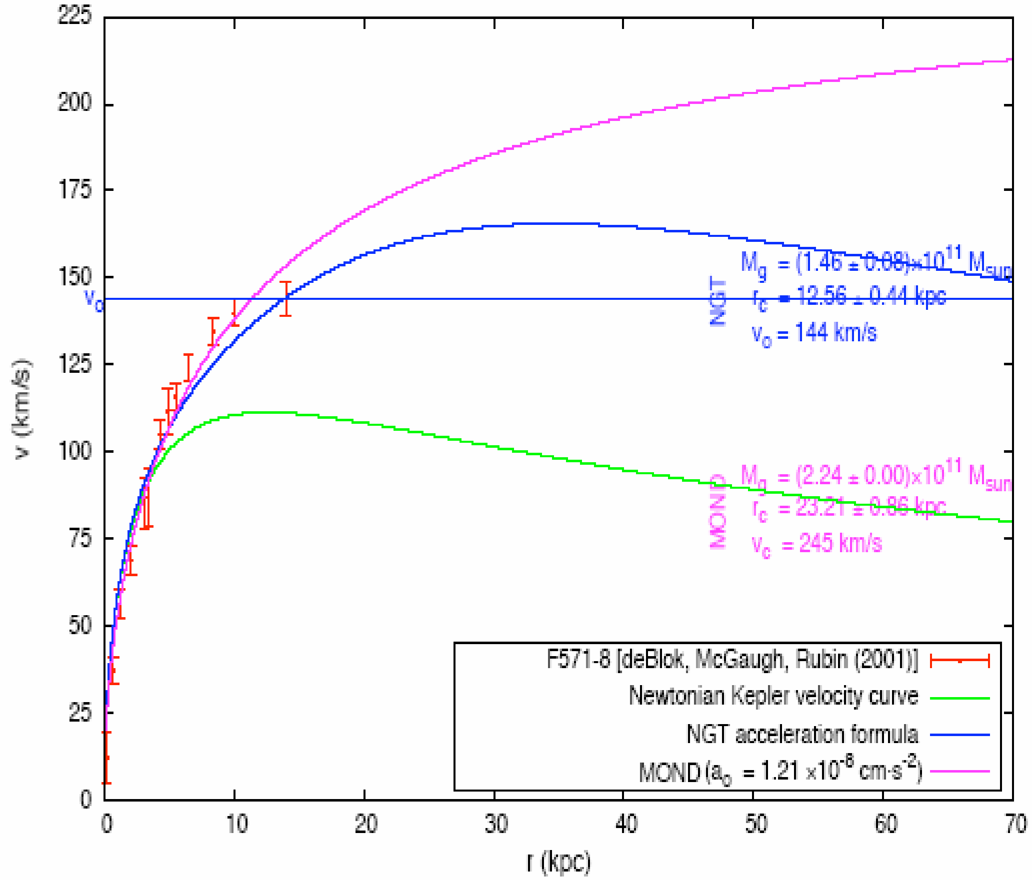

M_g = (1.46 ± 0.08)×10^11 M_{sun}
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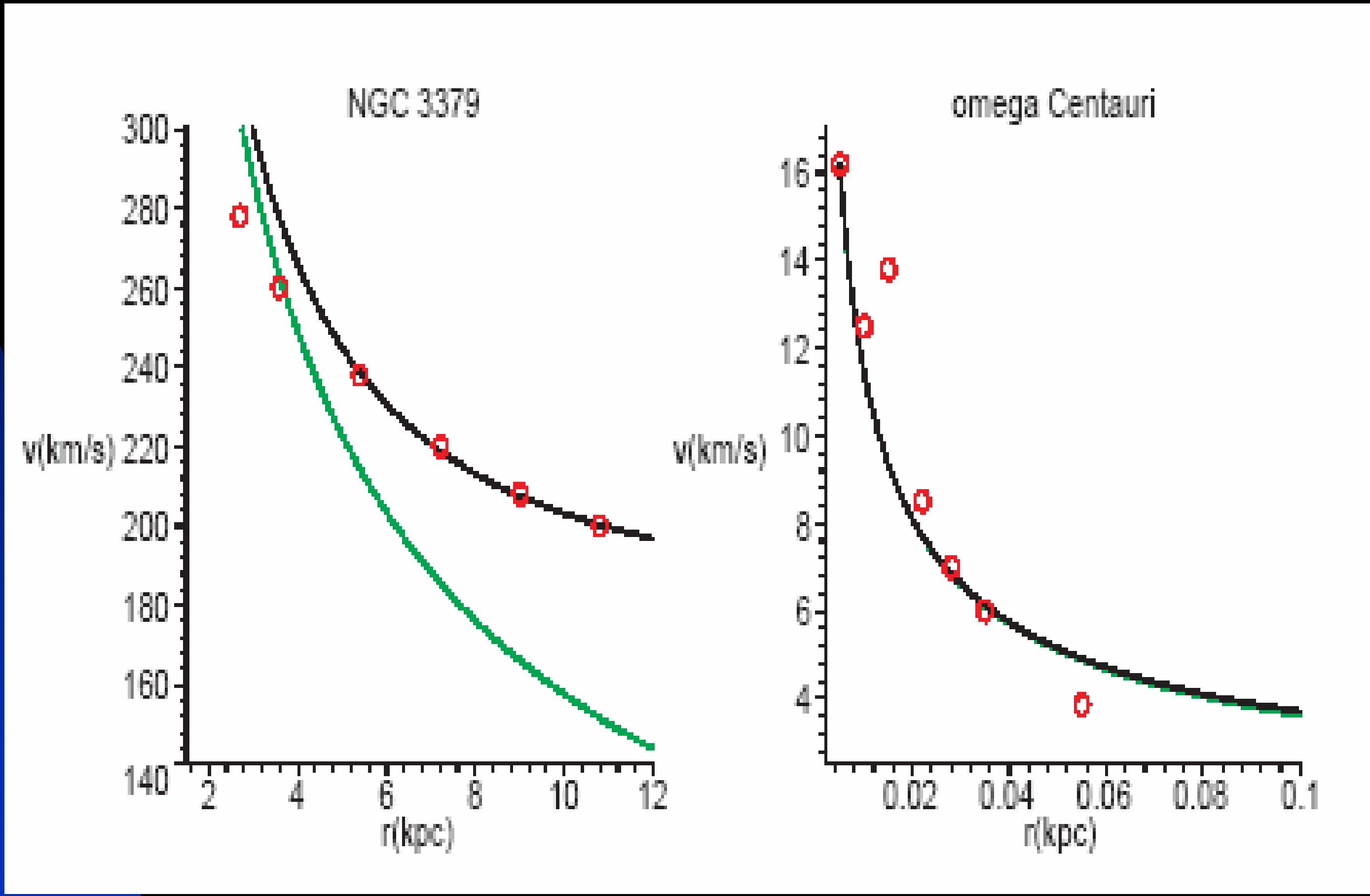
MOND (a_0 = 1.21 ×10^-8 cm·s^-2)
v (km/s)
r (kpc)


NGC 3379
300
280
260
240
v(km/s) 220
200
180
160
140
2
4
6
8
10
12
r(kpc)
omega Centauri
16
14
12
v(km/s) 10
8
6
4
0.02
0.04
0.06
0.08
0.1
r(kpc)

6. Galaxy clusters and lensing

The bending of a light ray as it passes near a massive system is to lowest order in v^2/c^2:

$$\theta = \frac{2}{c^2}\int |a^{\perp}| dz,$$

The light deflection is

$$\Delta = \frac{4GM}{c^2R} = \frac{4G_0\overline{M}}{c^2R},$$

$$\overline{M} = M\left(1+\sqrt{\frac{M_0}{M}}\right)$$

From the effective running G=G(r) we have

$$G(r) \to G_\infty = G_0\left(1+\sqrt{\frac{M_0}{M}}\right)$$

For a cluster $M_0 = 9.6\times 10^{14}\,M_\odot$ and a cluster mass $M_{\rm cl} \sim 10^{13}\,M_\odot$:

.

$$\left(\sqrt{\frac{M_0}{M}}\right)_{\rm cl} \sim 10.$$

7. Cosmology without dark matter

We assume that the universe is isotropic and homogeneous:

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right].$$

The Friedmann equations are

$$H^2(t) + \frac{k}{R^2(t)} = \frac{8\pi G\rho_M(t)}{3} + \frac{\Lambda}{3}$$

$$\ddot{R}(t) = -\frac{4\pi G}{3}[\rho_M(t) + 3p_M(t)]R(t) + \frac{\Lambda}{3}R(t).$$

For an FLRW universe, the averages of $A_{\mu\nu}$ and $F_{\mu\nu\lambda}$ vanish.

We have for the effective G=G_0Z obtained from the running of G=G(t) with time t. For a spatially flat universe k=0 and

$$\Omega_M(t) + \Omega_\Lambda(t) = 1$$

$$\Omega_M(t) = \frac{8\pi G\rho_M(t)}{3H^2(t)} \quad \Omega_\Lambda = \frac{\Lambda}{3H^2(t)}.$$

We impose physical restrictions on the running of G(t). At the time of BB nucleosynthesis $\Omega_B \sim 0.02 - 0.04$. From WMAP data at the surface of last scattering $\Omega_B \sim 0.04$. At $t_{BBN} \sim 10^{-7}$ yrs and at $t_{SLS} \sim 10^5$ yrs we have $G(t_{BBN}) \sim G_0$.

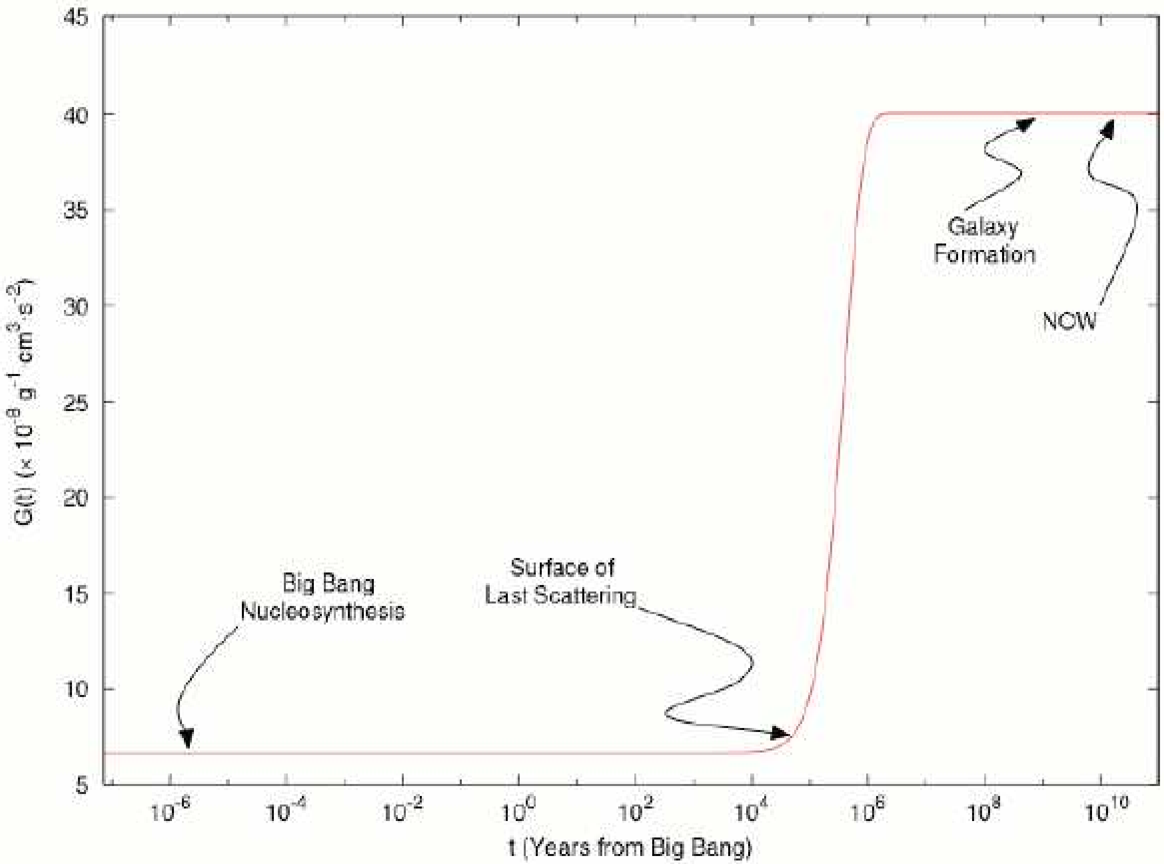
Since the time of the surface of last scattering (decoupling) $(z \sim 10^3)$, G(t) grows until the present time

$$G(t_{\rm now}) \sim 6G_0 \qquad \Omega_M \sim 6\Omega_B \sim 0.24$$

A bound on G changing with time from spacecraft measurements is

$$\frac{\dot{G}}{G} = (4 \pm 9) \times 10^{-13}\,{\rm yr}^{-1}$$

The form of the effective G(t) must be determined from the RG flow QG formalism and is model dependent.



G(t) (× 10⁻⁸ g⁻¹ cm³ s⁻²)
t (Years from Big Bang)
Big Bang
Nucleosynthesis
Surface of
Last Scattering
Galaxy
Formation
NOW

The cosmological constant $\Lambda(k)$ runs with k and time t. This corresponds to a quintessence scenario.

It is possible that the RG flow trajectories that lead to a large distance classical IR scenario can solve the cosmological constant problem, for these trajectories imply a small cosmological constant.

We have to perform a comparison of our baryon dominated matter era model with the WMAP power spectrum acoustical data to see whether a satisfactory fit can be obtained that compares well with the Lambda CDM model. The growth of large scale galaxies and clusters from an initially smooth background of fluctuations must also be investigated. Since the effective G increases after decoupling to a value G(t_now) ~ 6G_0, then the gravitational potential well will become deeper and allow for clumping of baryon matter to form galaxies without cold dark matter.

8. Conclusions

- We have developed a metric gravity theory coupled to a skew tensor field (MSTG) that leads to a modified Newtonian acceleration law that fits all available galaxy rotation curve data.
- We postulate a quantum gravity MSTG framework that leads to a running of G and the MSTG coupling constant in terms of an effective action. On the RG trajectory, we identify a regime of distance scales where solar system experiments are well described by GR.

- Strong infrared renormalization effects become visible at the scale of galaxies and cosmology.
- We have demonstrated that the RG flow running of G and MSTG cosmology can lead to a description of the universe that does not require dominant, exotic dark matter. Dark energy is described by a time dependent cosmological constant. Further work must show that the model can fit the WMAP power spectrum data.
- The RG flow trajectories can possibly lead to a solution of the cosmological constant problem.