Some useful relations:

Pressure & depth: \( p = p_{\text{top}} + \rho gh \)

Continuity: \( A_1 v_1 = A_2 v_2 \)

Bernoulli: \( p_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2 \)

Specific heat capacity: \( dQ = m c dT \)

Latent heat: \( Q = \pm mL \)

Heat current: \( H = \frac{dQ}{dT} = k A \frac{T_H - T_C}{L} \)

Average molecular energy per degree of freedom: \( \frac{1}{2} k_B T \)

1st lat of thermodynamic: \( \Delta U = Q - W \)

Heat in an isobaric process: \( Q = n C_P \Delta T \)

Heat in an isochoric process: \( Q = n C_V \Delta T \)

Work: \( dW = p \, dV \)

Ideal gas equation of state: \( p \, V = n \, R \, T \)
Engine efficiency: \( e = \frac{W}{Q_H} \)

Entropy: \( dS = \frac{dQ}{T} \)

Wave equation: \( \frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x,t)}{\partial t^2} \)

Speed of propagation of a wave on a string: \( v = \sqrt{\frac{T}{\mu}} \)

Doppler shift: \( f_L = \frac{\frac{v}{v} + v_L}{v + v_S} f_S \)

Intensity: \( I = \frac{\text{Power}}{\text{Area}} \)

Light reflection: \( \theta_r = \theta_a \)

Light refraction: \( n_a \sin \theta_a = n_b \sin \theta_b \)

Mirrors and thin lenses: \( \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \), \( m = \frac{y'}{y} = -\frac{s'}{s} \)

with \( f = \frac{R}{2} \) for mirrors and \( \frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \) for lenses

Diffractive interface: \( \frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \), \( m = \frac{y'}{y} = -\frac{n_a s'}{n_b s} \)

Phase difference in interference: \( \phi = 2\pi \Delta N \), with \( \Delta N = \) difference in # of wavelengths

For path difference: \( \Delta N = \frac{r_1 - r_2}{\lambda} \)

Constructive interference: \( \phi = 2n\pi \); Destructive interference: \( \phi = (2n + 1)\pi \)

Time dilation: \( T_{obs} = \gamma \cdot T_0 \) with \( T_0 = \) proper time

Length contraction: \( L_{obs} = L_0 / \gamma \) with \( L_0 = \) proper length

Relativistic Doppler Shift: \( f_{obs} = \frac{\sqrt{1+(u/c)}}{\sqrt{1-(u/c)}} f_{source} \) (for approaching source)

Lorentz transformations:
\[
\begin{align*}
  x' &= \gamma(x - ut) \\
  y' &= y \\
  z' &= z \\
  t' &= \gamma \left( t - \frac{ux}{c^2} \right)
\end{align*}
\]

Lorentz velocity transformations:
\[
\begin{align*}
  v'_x &= \frac{v_x - u}{1 - \left( \frac{v_x u}{c^2} \right)} \\
  v'_y &= \frac{v_y}{\gamma \left[ 1 - \left( \frac{v_x u}{c^2} \right) \right]} \\
  v'_z &= \frac{v_z}{\gamma \left[ 1 - \left( \frac{v_x u}{c^2} \right) \right]}
\end{align*}
\]
Problem #1

Sam is traveling from Earth to Mars on a spaceship that moves at speed \( u \) with respect to Earth. In his reference frame Sam measures the distance between the Earth and Mars to be equal to \( d \), and the spaceship to be of length \( L \).

1. What is the distance from Earth to Mars as measured by an observer on Earth?
2. What is the length of the spaceship as measured by an observer on Earth?
3. How long is Sam’s trip from his point of view?
4. How long is his trip from the point of view of an observer on Mars?
5. Halfway through his journey Sam sends a rocket back to Earth with speed \( -v \) compared to the spaceship. How fast is the rocket moving in the Earth reference frame?
6. If \( v > u \), how long will the rocket take to reach The Earth in the Earth’s reference frame?

NOTE: you can leave your answers in terms of \( \gamma \), as long as you first explicitly write the expression for \( \gamma \).
Problem #2

Light with frequency $f$ is emitted in air from a source $S$. Light can get reflected back to the source by mirror 1, a distance $y$ from the source or by mirror 2, a distance $x$ from the source (see figure).

a) What is the wavelength of the light in air?
b) What is the phase difference $\phi$ between the light reaching the source back after reflection from mirror 1 and that reaching it after reflection from mirror 2?
c) Using part b, write the condition for constructive interference between the two light beams.
d) A block of glass, with length $L$ and index of refraction $n$ is placed between the source and mirror 1. What is the wavelength of the light in the glass?
e) What is the additional phase shift $\phi'$ introduced by the glass?
f) Write the condition for constructive interference in this case (i.e., when you have the path difference AND the glass).
Problem #3

A hot air balloon is filled with $n$ moles air (density $\rho_{\text{air}}$). The initial pressure of the balloon is the atmospheric $p_o$, and the temperature is the room temperature $T_o$. Assume that the air can be considered as a diatomic ideal gas.

a) What is the initial volume of the balloon?

The gas inside the balloon is heated at constant pressure until the volume of the gas is doubled.

b) What is the final temperature of the gas?

c) What is the heat transfer to the gas?

d) What is the change in Entropy of the gas?

e) Because of the expansion, the density inside the gas is reduced and the balloon can float in air. What must be the mass $m$ of the balloon so that its altitude does not change (i.e., the balloon is in equilibrium)?
STUDENT NAME: ______________________________