Some useful relations:

Density: \( \rho = \frac{m}{V} \)

Pressure & depth: \( dp = \rho g dy \); if \( \rho \) is constant: \( p = p_o + \rho gh \)

Continuity: \( A_1 v_1 = A_2 v_2 \)

Bernoulli: \( p_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2 \)

Volume expansion: \( dV = V_0 \beta dT \)

Specific heat capacity: \( dQ = mc dT \)

Latent heat: \( Q = \pm mL \)

Heat transfer: \( \frac{dQ}{dt} = kA \frac{T_H - T_C}{L} \)

Ideal gas equation of state: \( pV = nRT \)

Quadratic equation: \( ax^2 + bx + c = 0 \) \( \Rightarrow \) \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)
Problem #1

A large, sealed container with area $A$ and height $L$ is filled with water (density $\rho$) up to level $y$. The remainder of the container is filled with $n$ moles of air at temperature $T$. A small hole with area $\sigma$ is punched at the bottom of the container to let water come out.

a) Calculate the pressure of the air in the container when the water level is $y$.

b) Calculate the speed at which water comes out of the container.

c) As water comes out of the container, the water level drops. At some point, however, water stops coming out of the container. Find the level $y_o$ at which this happens.

\[
a) \quad \frac{nRT}{A(L-y)} \\
b) \quad \sqrt{\frac{2 \left( \frac{nRT}{A(L-y)} \rho_o + \rho y \right)}{\rho}} \\
c) \quad \frac{1}{2} \left( -\frac{\rho_o + \rho g L}{\rho g} + \sqrt{\left( \frac{\rho_o + \rho g L}{\rho g} \right)^2 - 4 \left( \frac{\rho_o L}{\rho g} - \frac{nRT}{\rho g A} \right)} \right).
\]
Problem #2

Two rods with same length $d$ and same sectional area $A$ are made of copper (thermal conductivity $k_c$) and aluminum (thermal conductivity $k_a$). The two rods are connected in series between two systems, one consisting of a mass $m$ of ice at $T_1=0^\circ C$ (latent heat $L$), the other maintained at a constant temperature $T_2>T_1$ (see figure). Assume that there is no dissipation of heat to the surroundings.

a) Calculate the temperature $T_o$ of the joint between the two rods.
b) Calculate the heat transferred per unit time between the two systems.
c) Calculate the mass of ice melting per unit time ($dm/dt$).
d) Find the time necessary to melt all the ice.

\[ a) \frac{k_c T_1 + k_a T_2}{(k_c + k_a)} . \]
\[ b) k_c k_a \frac{A (T_2 - T_1)}{A (k_c + k_a)} . \]
\[ c) \frac{1}{L} k_c k_a \frac{A (T_2 - T_1)}{A (k_c + k_a)} . \]
\[ d) \frac{Lm d (k_c + k_a)}{A k_c k_a (T_2 - T_1)} . \]
Problem #3

Let’s assume that the air in our atmosphere can be considered an ideal gas with molar mass $M$ and constant temperature $T$.

a) Derive a relation between density and pressure of an ideal gas $\rho = \rho(p)$.

b) Use the result from part a) to find a relation between the change in pressure $dp$ in the atmosphere and the change in altitude $dy$. Remember that pressure decreases as altitude increases.

c) If the pressure at sea level is $p_o$, use the result from part b) to find an expression for the atmospheric pressure as a function of altitude $y$.

\[
\begin{align*}
\text{a)} & \quad \frac{pM}{RT} \\
\text{b)} & \quad -\frac{pM}{RT} g dy \\
\text{c)} & \quad p_o e^{-\frac{Mg}{RT}y}
\end{align*}
\]