Some useful relations:

Density: \( \rho = \frac{m}{V} \)

Pressure & depth: \( p = p_{\text{top}} + \rho gh \)

Continuity: \( A_1 v_1 = A_2 v_2 \)

Bernoulli: \( p_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2 \)

Volume expansion: \( dV = V_0 \beta dT \)

Specific heat capacity: \( dQ = mc dT \)

Latent heat: \( Q = \pm mL \)

Heat current: \( H = \frac{dQ}{dT} = kA \frac{T_H - T_C}{L} \)

Ideal gas equation: \( pV = nRT \)

Equipartition of energy: Average kinetic energy \( \frac{1}{2} k_B T \) per degree of freedom
Problem #1

The coefficient of volume expansion of a metal, instead of being constant, changes with temperature according to the following relation:

\[ \beta = a + bT^2, \]

Where \( a \) and \( b \) are constants.

a) If the volume of a block of such metal at temperature \( T_0 \) is \( V_0 \), calculate the volume of the same block when the temperature is raised to a value \( T_1 > T_0 \). Write your answer in terms of \( a, b, T_0, V_0, \) and \( T_1 \).

Write your answers to questions b) through e) in terms of \( T_0, V_0, T_1, V_1, \rho_0, \) and \( c \) [i.e., don’t worry about question a) in order to answer questions b) through e)].

b) If the mass density of the block at temperature \( T_0 \) is \( \rho_0 \), what is the mass of the block?

c) What is the density of the block at temperature \( T_1 \)?

d) If the specific heat capacity of the metal is \( c \), how much heat transfer \( Q \) is necessary to change the temperature from \( T_0 \) to \( T_1 \)?

e) Is that heat transfer positive or negative? Explain.
Problem #2

An apparatus to measure the speed of water is shown in the figure. It consists of a horizontal pipe with radius $R_1$. The pipe has a constriction at the right end, open to air, with radius $R_2=R_1/2$. The pipe is connected to another vertical pipe of length $L$, closed at the end and filled with air.

When the speed of the water out of the constriction is $v_2$, the level of the water in the vertical pipe is $h_1$. The density of the water is $\rho$.

a) What is the speed of the water in the pipe under these conditions?
b) What is the pressure in the horizontal pipe?
c) What is the pressure in the air trapped in the vertical pipe?
d) When the water flow in the pipe is changed, the level in the vertical pipe changes to a value $h_2$. What is the new pressure of the water in the pipe? Treat the air in the vertical pipe as an ideal gas.
Problem #3

a) A cubic container of volume $V$ contains a polyatomic ideal gas at pressure $p$. According to the principle of equipartition of energy, what is the total kinetic energy of the gas? Write your result in terms of $p$ and $V$.

b) The state of the gas is represented by point 1 on the p-T phase diagram shown in the figure. What happens if the temperature of the gas is reduced at constant pressure (draw the process in the figure and explain)?

c) What happens if instead the temperature is reduced at constant volume (draw the process in the figure and explain)?

d) What happens if the pressure is increased at constant temperature (draw the process in the figure and explain)?

e) In the case of point d), how does the total energy (kinetic plus potential) of the system change during the process? Explain.