Some useful relations:

Wave equation: \( \frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x,t)}{\partial t^2} \)

Doppler shift: \( f_L = \frac{v + v_L}{v + v_S} f_S \) (remember sign convention)

Intensity: \( I = \frac{\text{Power}}{\text{Area}} \)

For periodic wave: \( I = \frac{1}{2} A^2 w^2 \sqrt{\rho B} = \frac{p_{\text{max}}^2}{2 \sqrt{\rho B}} \)

Speed of propagation of sound in ideal gas: \( v = \sqrt{\frac{\gamma RT}{M}} \)

Light reflection: \( \theta = \theta_a \)

Light refraction: \( n_a \sin \theta_a = n_b \sin \theta_b \)
Question #1

Consider an organ pipe of length $L$ open at the left end and closed at the right end.

1. Do you have a displacement node or antinode at the left end of the pipe? At the right end?

2. Using the axes below, draw the amplitude $y$ of the sound waves produced in the pipe as a function of the distance $x$ along the pipe for the first three modes of the pipe.

3. From your drawings, derive expressions for the wavelengths of the first three modes of the pipe as a function of the length $L$.

4. From your answer to part 2, derive an expression for the frequency of the first three modes (assume that the speed of sound waves in air is $v$).

5. If the intensity of the sound from the pipe at a distance $d_o$ is 30 dB (decibels), how far away from the pipe can you hear its sound (remember that the threshold for human hearing is about 0 dB).

6. A second pipe, identical to the first one, is played at the same time as the first one. However, the second pipe is colder than the first one and the sounds from the two pipes produce a beat with beat frequency $\Delta f$. What is the difference in temperature $\Delta T$ between the two pipes? Consider the air as an ideal gas.

NOTE: You may find useful the expression used in class, for a generic function $y(x)$ and small changes in $x$: $\Delta y = \frac{dy}{dx} \Delta x$.

![Diagram of the first three modes of the pipe](image)
Question #2

A horizontal light beam is incident on a prism as in the figure. The angles \( A \) of the prism are 45°, the index of refraction of the prism is \( n \), while the index of refraction of air is approximately 1.

1. Draw the light beam as it travels through the prism.
2. Calculate the angle between the initial direction of the light beam and its direction after the first diffraction.
3. Calculate the angle between the initial direction of the light beam and its direction after the second diffraction.
4. If the wavelength of the light in air is \( \lambda_o \), what is its frequency in air?
5. What are its frequency and wavelength inside the prism?
6. Now assume that, instead, the beam is composed of white light and that the index of refraction of the prism is dependent on the wavelength. In particular, assume that the index of refraction is greater for longer \( \lambda \). What happens to the beam after the two diffractions? Explain in no more than a few words.
7. Which color of the rainbow comes out of the prism toward the positive y direction and which one comes out toward the negative y direction?
A truck and a police car are approaching each other with relative speed $v$. The siren of the police car is producing a sound with frequency $f_0$.

1. Is this information sufficient to determine the frequency of the sound heard by the driver of the truck? Explain in a few words.
2. If the police car is traveling at speed $v_p$, what is the frequency of the sound heard by the driver of the truck?
3. What is the frequency of the sound heard by the driver of the truck when the two vehicles have passed each other?
4. Now suppose the sound produced by the police car is reflected from a wall that is in front of the car. The driver of the police car therefore hears the beats produced by the combination of the direct siren sound and the sound reflected from the wall. What is the beat frequency?
5. The driver of the police car decides to increase the volume of the siren, so that the intensity of the sound wave is doubled. What are the change in the amplitude and pressure amplitude of the sound wave?