Problem #1

Two identical point charges $q$ are placed on the $y$ axis at points $y=a$ and $y=-a$ respectively.

a) Find the electric potential for all points on the $x$-axis.

b) Find the point on the $x$-axis where the electric potential is maximum.

c) A particle with negative charge $-q_o$ and mass $m$ is placed at rest at $x=\infty$. Calculate the potential energy of the particle along the $x$-axis and explain in one short sentence why the point of maximum electric potential corresponds to the minimum potential energy of the particle.

d) Calculate the speed of the particle when it reaches the point of minimum potential energy.
**Problem #2**

In the circuit shown below the capacitor is initially uncharged and the switch S open. At time $t=0$ the switch is closed.

a) Find the current $I$ through the capacitor and the charge $Q$ on it immediately after the switch is closed.

b) Find the current $I$ through the capacitor and the charge $Q$ on it a long time after the switch is closed.

c) Find the energy stored in the capacitor a long time after the switch is closed.

d) Find the power provided by the battery a long time after the switch is closed.

Explain your reasoning.
Problem #3

A cylindrical rod with diameter $D$ and length $L$ is made of two different materials back to back. Half of the rod has resistivity $\rho_1$, the other has resistivity $\rho_2$ (see figure). A battery with e.m.f. $\mathcal{E}$ and internal resistance $r_o$ is connected to the rod.

a) Calculate the total resistance of the rod.

b) Calculate the potential difference across the rod.

c) Calculate the potential of the middle point of the rod with respect to its right end (points $a$ and $b$ in the figure).

d) Calculate the electric field $\overrightarrow{E}$ inside the right half of the rod (assume it is uniform).
Some useful relations:

\[ \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}, \quad \vec{E} = \frac{1}{4\pi\varepsilon_0} \sum q_i r_i^{-2} \hat{r}_i, \]

\[ \vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r^2} \hat{r}, \]

\[ \vec{F} = q_o \vec{E} \]

Dipole: \( \vec{\tau} = \vec{p} \times \vec{E}, \quad U = -\vec{p} \cdot \vec{E} \)

Gauss Law: \( \phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \)

\[ V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}, \quad V = \frac{1}{4\pi\varepsilon_0} \sum q_i r_i^{-1}, \quad V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r} \]

\[ \vec{E} = -\nabla V = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k} \]

\[ \Delta U = q \Delta V, \quad V_{ab} = \int_a^b \vec{E} \cdot d\vec{l} \]

\[ A_{sph} = 4\pi r^2, \quad V_{sph} = \frac{4}{3} \pi r^3, \quad dV_{sph} = 4\pi r^2 dr \]

\[ A_{cyl} = 2\pi r L, \quad A_{sph} = 4\pi r^2 \]

\[ dq = \lambda dl = \sigma dA = \rho dV \]

Capacitors: \( C = \frac{Q}{V} \)

Parallel plates: \( E = \frac{\sigma}{\varepsilon_0} \)

Capacitors in parallel: \( C_{eq} = C_1 + C_2 + ... \)

Capacitors in series: \( \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... \)

Energy: \( U = \frac{1}{2} QV \)

Energy density: \( u = \frac{1}{2} \varepsilon E^2 \)

Dielectrics: \( \varepsilon = K\varepsilon_0 \)

Current: \( I = \frac{dQ}{dt}; \quad J = I/A \)

Resistors: \( R = \frac{V}{I}; \quad \rho J = \vec{E}; \quad R = \rho L/A \)

Resistors in series: \( R_{eq} = R_1 + R_2 + ... \)

Resistors in parallel: \( \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... \)

Power: \( P = VI \)

Kirchhoff rules: \( \sum I = 0 \) - node rule

\[ \sum V = 0 \] - loop rule