Name: ____________________________

Signature: _________________________

I.D. #: ____________________________

Instructions:
• Use only the front side of the sheets for the exam.
• Use the back side for intermediate calculations and everything that you don’t want the instructor to grade.
• If you need more sheets of paper ask them to the instructor, don’t use your own.
• If you cannot solve a part of a problem and you need that result for the rest of the problem, solve the remaining of the problem using symbols instead of numerical values.
Part I

Part 1:

a) Use Schrödinger equation to derive the time dependent wavefunction of an electron moving with energy $E$ toward the positive $x$-axis in a constant potential energy $U(x) = U$ for $-\infty < x < \infty$.

b) Is that an eigenfunction of the operator momentum? Explain.

c) What is the DeBroglie wavelength of the electron in terms of the total energy $E$ and the potential $U$?

Solution:
A particle is described by the wavefunction:

\[ \psi(x) = \begin{cases} 
C \cdot (4x - x^2) & \text{if } 0 \leq x \leq 4 \\
0 & \text{elsewhere}
\end{cases} \]

where $x$ is in nm and $C$ is a constant.

**a)** What is the value of $C$ that normalizes the probability associated with the wavefunction?

**b)** Calculate the expectation and uncertainty for the particle momentum.

**Solution:**
Part III

An electron with energy $E < U$ is incident (from left to right) on the step barrier shown in the figure. [$U(x) = 0$ for $x < 0$, $U(x) = U$ for $x > 0$]

I                                  II

U

$x=0$

a) What is the expression for the wave function in region I? Explain.

b) What is the expression for the wave function in region II? Explain.

c) What are the conditions that a wavefunction must satisfy?

d) Write the relations between the parameters of the two functions. Note: write the system of equations, don’t solve it.

e) If we measure the particle position, can we find it in region II? Explain.

Solution:
Part IV

a) What is the wavelength of the photon emitted by an electron confined in a box of length $L=10^{-10} \text{ m}$ going from the $n=4$ to the ground level?

b) Draw the wavefunction and probability density of a particle in the $n=3$ level of the following potential:

$$U(x) = \begin{cases} 
0 & \frac{L}{2} < x < \frac{L}{2} \\
U & \text{elsewhere}
\end{cases}$$

NOTE: assume for the particle energy that $E<U$.

Solution: