Errata and addenda to

Equilibrium and Non-equilibrium Statistical Mechanics
by
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À propos : Everyone who ever published an article has faced the conundrum that errors tend to only become visible when the paper appears in print; it is not any different with publishing a book, particularly one of this length. Since this book was prepared in camera-ready form, the responsibility rests solely with the author. We apologise for inconveniences that ‘essential errors’ may have caused; also for the many instances of wrongly referenced equations. Our main solace: the Publisher will incorporate the changes noted below in a second, revised printing.

Errors, noticed while we read, re-read, and taught graduate courses from this textbook, along with various improvements that presented themselves upon renewed reflection, are listed by chapter, page, alinea and ‘line’. Lines are counted from the left margin by paragraph, irrespective of disruption by display equations.

Chapter I

24, Eq. (1.6-30): Last factor on lhs should read \((N-N_1-\ldots N_{K-2})!/N_{K-1}!(N-\Sigma N_j)!!\)

27, §2, 2: Substituting \(y^2 = \alpha x^2\).

Chapter II

54, §2, 3: Section 1.5
55, §3, 6: \(\Delta \Omega = h^{\Omega N} N!\)
57, §2, 4: Comparing (2.5-3) and (2.5-3’)
64, Problem 2.6: where \(L\) is the \textit{superoperator} … defined by \(L K = (1/h)[\hat{H}, K]\).

Chapter III

73, §3, 6: cf. (2.4-15).
76, Eq. (3.5-15): lhs should read \(S(T,N)/k_B\)
§3, 3: where \(\alpha = h\omega/k_B T\).
79, Eq. (3.5-30): add two minus signs, \(-\partial \zeta / \partial T\)_{V,N} and \(-\partial \zeta / \partial V\)_{T,N}
83, Eq. (3.6-14): \((\partial^2 G / \partial T^2)_{E_{N,I},A} = \ldots\)
§2, 4: of gas and liquid balance,…(omit non-applicable symbols)
86, §1, 4: where on the rhs, strictly speaking, \(a = \langle \eta | a | \eta \rangle\).
Chapter IV

94, Eqs. (4.1-1) and (4.1-2) should have $E^c$ instead of $E_c$.

118, §1, 3: after $k \ll N$ add footnote 18c:

18c For the general term we have:

$$\frac{1}{k!} \left( \frac{N}{2} \right)^k \approx \frac{1}{k!} \left( \frac{Nb_2}{V} \right)^k \approx \left( \frac{N}{k} \right)^k \left( \frac{Nb_2}{V} \right)^k.$$  (4.4-51')

123, Eq. (4.5-15): $= -k_B T \left[ (1 - n \alpha) \zeta_0 + n \ln K(\zeta_0) \right]$. 

130, §2, 8: add footnote 31a:

$$f^{(n)}(z_0) = \sum_{n=1}^{\infty} \frac{a_n}{(z - z_0)^n} dz;$$ for a Maclaurin series $f(z) = \sum a_n z^n$, $a_n = \frac{1}{2\pi i} \oint \frac{f(z)}{z^n} dz$.

130, Eq. (4.7-14): $I^2 b_1 = \frac{1}{2\pi i} \oint \frac{\zeta(dw/d\zeta)}{\zeta^{n+1}} d\zeta = \frac{1}{2\pi i} \oint \frac{\exp(I\Sigma_k \beta_k \omega^k)}{\omega^l} d\omega$.  (4.7-14)

131, §1, 7: add, below, depicting two-body, three-body, four-body interactions, etc.

137, Fig. 4-13: the lettering of the straight lines should read: slope $= k_B T_z / pJ_0$, etc.

146, Fig. 4-17: in the ordinate replace $F$ by $\Phi$.

168, §1, 1: plane except at the poles $z = 0, -1, -2, \ldots$

Chapter V

171, Fig. 5-1: lettering between the boxes, $\delta = A_0$

Chapter VI

207, first Eq. (6.3-8): add $d\omega$ before the slash.

second Eq. (6.3-8): relabel (6.3-9); Eq. (6.3-9), relabel (6.3-9')

211, Eq. (6.5-3): add $d^3q$ in $[\cdots]$;

215, Fig. 6-3(b): the $\xi$ most to the right should be replaced by $\zeta$.

217, §3, 7:8: replace by a new paragraph,

We shall introduce the characteristic temperature $\theta_{rot}$ by $k_B \theta_{rot} = \hbar^2 / 2I$ and the parameter $\sigma$ by $\sigma = \theta_{rot} / T$. For very small $\sigma$ the series (6.7-14) is rapidly convergent. For moderately small values, $\sigma (\leq \frac{1}{2})$, the convergence is poor; we will find another series to be more useful. Generally, rotator partition functions can be expressed in …

227, Problem 6.7, part (d): replace by

(d) Let $L$ be large; find again $P/I$ and confirm the barometric height formula.

Chapter VII

255, Eq. (7.8-1): replace $H$ by $J$.  

Chapter VIII

264. Fig. 8-1: A small tangent segment dS should be drawn \( \perp \) to the dashed line ---

266. Eq. (8.9-2): last factor \((n_1^2 + n_2^2 + n_3^2)\). 274, §1, 8: rather than \( \propto T^{3/2} \). Omit line 9.

286, Fig. 8-10: Place the “\((2\times)\)” behind the lines TO and TA.

287, Fig. 8-12: the three dashed lines emanating from point O should be parabolically curved.

289, Eq. (8.6-2): \( e_{\alpha\beta} = \partial u_\beta / \partial x_\alpha = \partial u_\beta / \partial x_\alpha \), or \( e = \text{Grad} u \).

305, §2, 1-7: replace by

We will now find the response function \( \varphi(r) \equiv \delta m(r) / \delta h(r) \) and the

generalized susceptibility \( \chi = \int d^3 r \varphi(r) \). Since there is no integral equation for

\( m(r) \) in terms of \( h(r) \), we must do the functional differentiation in (9.2-13) for

this case implicitly. This is carried out by assuming that we have a small perturbation centred on \( r' \), i.e. we set \( h(r) = h_0(r) + h_1 \delta(r - r') \), with response \( m(r) = m_0(r) + h_1 \varphi(r) \). Substituting into (9.2-13) and retaining only terms

linear in \( h_1 \), one obtains

\[
\begin{align*}
\frac{h_0(r) + h_1 \delta(r - r')}{ss} &= a_2 m_0(r) + a_2 h_1 \varphi(r) + a_4 [m_0(r)]^3 \\
&\quad + 3a_4 [m_0(r)]^2 h_1 \varphi(r) + \ldots - b \nabla^2 m_0(r) - b h_1 \nabla^2 \varphi(r). 
\end{align*}
\]

(9.2-13')

Cancelling the steady-state terms and dividing out \( h_1 \), we find for \( \varphi(r) \):

\[
\nabla^2 \varphi - \xi^{-2} \varphi = -b^{-1} \delta(r - r'),
\]

(9.2-14)

with

\[
\xi = \sqrt{\frac{b}{a_2 + 3a_4 m_0^2(r)}} = \begin{cases} 
b / a_{20}(T - T_c), & T > T_c, \\
b / 2a_{20}(T_c - T), & T < T_c. 
\end{cases}
\]

(9.2-14')

316, Eq. (9.4-22): upper line should read

\[
C_V = x_g (V_g) + x_T (T) + \left[ \frac{\partial \tilde{V}_g}{\partial T} \right]_{\text{coex}} \left[ T \left( \frac{\partial S_g}{\partial V_g} \right)_{T} - T \frac{dP}{dT} \right]
\]

317, Eq. (9.4-26), second part should be \( + x_i \left[ C_V - T \left( \frac{\partial \tilde{V}_l}{\partial T} \right)_{\text{coex}} \left( \frac{\partial P}{\partial V_l} \right)_{T} \right] \).

(9.4-26)
Chapter X

340, §1, 3: result is $\text{tr} [...]^N$, where ‘tr’ denotes the matrix trace.\(^9a\) Add footnote\(^9a\):

\(^9a\) Henceforth tr denotes the trace of a matrix, while Tr will be reserved for the trace of an operator.

346, §1, 28-29: \(J_1/k_B T\); the points to the left of the line indicate high temperatures (disordered state) and those to the right low temperatures (ordered state).

Chapter XI

380, Eq. (11.2-25):

\[
c_{m+1} = -c_1, \quad c_{m+1}^\dagger = -c_1^\dagger, \quad \text{(anti-periodic) for } n \text{ even}, \\
c_{m+1} = c_1, \quad c_{m+1}^\dagger = c_1^\dagger, \quad \text{(periodic) for } n \text{ odd}.
\]

383, Eq. (11.2-42), last line: \(V_q = (V_{ij})^{2} V_{2q} (V_{ij})^{2}, \quad V_{0 \text{ or } \pi} = V_{10 \text{ or } \pi} V_{20 \text{ or } 2\pi}\).

383, Eq. (11.2-40): there is a minus sign in front of \(2K^+_q\)

\[
V_{10} = \exp[-2K^+_1(c_0^c c_0 - \frac{1}{2})], \\
V_{1\pi} = \exp[-2K^+_1(c_\pi^c c_\pi - \frac{1}{2})], \quad (11.2-40)
\]

384, Eq. (11.2-45) \(\rightarrow\) (11.2-45a):

\[
c_q^c c_q + c_q^\dagger c_{-q} = 2b_q^1 b_q \rightarrow 2b_q^1 b_q \equiv \beta_{q z} + 1,
\]

§2, 12: \((\beta_q^z)^2 = 1\) since…; label the last two equation as (11.2-45b), (11.2-46)

385, Eq. (11.2-47), first line \(V_{2q} = \exp\left[2K_2^q[(\beta_{q z} + 1)\cos q + \beta_{q z} \sin q]\right]\)

388, §2, 3-4, Eq. (11.3-3): We now turn to the case of odd total fermion occupancy.

We need the eigenvalues for the operators in (11.2-40); remembering that the 0-state is occupied and the \(\pi\)-state is empty, we find

\[
\begin{align*}
V_{10} &= e^{-K^+_1} \\
V_{1\pi} &= e^{K^+_1} \\
V_{20} &= e^{2K^+_1}, \\
V_{2\pi} &= 1.
\end{align*}
\]

393, Eq. (11.3-27): lhs reads \(\hat{u}(T) = (d/d\beta)(\hat{f}(0, T)) = \ldots\)

395, §2, 8: the sum of the squares, as well as the short-range correlations, disappear


412, Problem 11.1, Eq. (1) should be amended to read

\[
V^{1/2}_2 = \sqrt{A} \exp\left[\frac{1}{2} K^{+}_s \sigma_z\right] = \sqrt{A}\left[\cosh \left(\frac{1}{2} K^{+}_s\right) \mathbf{1} + \left(\sinh \left(\frac{1}{2} K^{+}_s\right) \sigma_z\right)\right], \\
V_1 = \exp(\beta \hbar \sigma_z) = (\cosh \beta \hbar) \mathbf{1} + (\sinh \beta \hbar) \mathbf{1} + (\exp \beta \hbar) \sigma_z.
\]

Problem 11.3 should be replaced by:

11.3. Obtain the forms of the four particular diagonal operators given in Eq.(11.2-40).

While these are all the necessary and recommended changes for Chapter XI, the chapter can be improved in presentation if two types of changes are made additionally. On p. 382 one might do the splitting into positive and negative \(q\) somewhat later. On p. 399 the results are obtained faster if the sign of the \(q\)’s is changed for the annihilation operators, rather than for the creation operators. We thus offer new optional pages 382, 383, 384 and 399.
Hence we have, changing $j' \to j$,
\[
c_j = \frac{e^{-i\pi j/4}}{\sqrt{M}} \sum_q c_q e^{iqj}; \quad \text{likewise} \quad c_j^+ = \frac{e^{i\pi j/4}}{\sqrt{M}} \sum_q c_q^* e^{-iqj}. \tag{11.2-32}
\]

The same lemmas can be used to show that the $c_q$'s satisfy the fermion anticommutation rule
\[
[c_q, c_q^+] = \delta_{q,q'}; \tag{11.2-33}
\]
the proof is left to the reader.

We must fix the store of $q$-values, such that the b.c. (11.2-25) are satisfied. We thereto set,
\[
q = k\pi / M, \quad \text{with} \quad \begin{cases} k = \pm 1, \pm 3, \pm 5, \ldots (M-1), & \text{for } n \text{ even}, \\ k = 0, \pm 2, \pm 4, \ldots + M, & \text{for } n \text{ odd}. \end{cases} \tag{11.2-34}
\]
(we assumed without undue restriction that $M$ is even). In each case there are $M$ values of $q$, like in the first Brillouin zone of a solid; note that for $n$ odd we omitted the value $k = -M$, corresponding to $q = -\pi$.

We must now transform the operators $V_1$ and $V_2$. The exponential in $V_2$ gives
\[
\sum_j (c_j^+ - c_j)(c_{j+1}^+ + c_{j+1})
\]
\[
= \frac{1}{M} \sum_{q,q'} \sum_j \left( e^{i\pi j/4} c_q^* e^{-iqj} - e^{-i\pi j/4} c_q e^{iqj} \right) \left( e^{i\pi j/4} c_q^* e^{-iqj} - e^{-i\pi j/4} c_q e^{iqj} \right)
\]
\[
= \frac{1}{M} \sum_{q,q'} \sum_j \left\{ i c_q^+ c_{-q} e^{-iqj} - e^{-iqj} + c_q c_q^* e^{-iqj} - e^{-iqj} + i c_q c_{-q} e^{-iqj} - e^{-iqj} \right\}
\]
\[
= \sum_q \left\{ i c_q^+ c_{-q} e^{-iq} + c_q^+ c_q e^{-iq} \right\}; \tag{11.2-35}
\]
we used here Lemma (A) and $c_q^+ c_q + c_q^* c_q = 1$. Be it further noted that the sum involving $\exp(-iq)$ is to be omitted.\(^{13}\) We now split into two sums, $\Sigma_{q>0}$ and $\Sigma_{q<0}$; in the latter we change $q \to -q$. [The terms with $q = 0$ and $q = \pi$, left out in this procedure for odd $n$, will be dealt with later.] We thus obtain
\[
(11.2-35) \to \sum_{q>0} \left\{ i c_q^+ c_{-q} e^{i\pi} + c_q^* c_q (e^{i\pi} - e^{-i\pi}) + ic_q c_{-q} e^{-i\pi} \right\}
\]
\[
+i c_{-q} c_q^* e^{-i\pi} + c_{-q}^* c_q (e^{i\pi} - e^{-i\pi}) + ic_{-q} c_q e^{i\pi} \right\}. \tag{11.2-36}
\]
\(^{13}\) By Lemma (B) we have $\Sigma_{q>0} \exp(iq) = 0$, which holds for both sequences in (11.2-34). For the odd case, we have in addition $\Sigma_{q=0,\pi} \exp(iq) = \Sigma_{q=\pi} \exp(i\pi) = 0$, since the $q = 0$ and $q = \pi$ terms cancel.
By changing the first and last term with the anticommutation rules, we then easily arrive at:

\[(11.2 - 36) \rightarrow 2 \sum_{q>0} \left[ \cos q \left( c_q^+ c_q + c_{-q}^+ c_{-q} \right) + \sin q \left( c_{-q}^+ c_{q} + c_q^+ c_{-q} \right) \right]. \tag{11.2-37}\]

We now note that operators with different wave numbers commute; this entails an enormous simplification, since the various \(q\)-contributions of the exponent can be written as a product of transfer-operator factors

\[V_2 = \prod_{q>0} V_{2q}, \text{ with} \]

\[V_{2q} = \exp \left\{ 2K_2 \left[ \cos q \left( c_q^+ c_q + c_{-q}^+ c_{-q} \right) + \sin q \left( c_{-q}^+ c_{q} + c_q^+ c_{-q} \right) \right] \right\}. \tag{11.2-38}\]

The \(c\)-operators are now uncoupled, except for the mixing of \(+q\) and \(-q\). We leave it to the reader to obtain the result for \(V_1\) by a similar procedure,

\[V_1 = (2 \sinh 2K_1)^{M/2} \prod_{q>0} V_{1q}, \text{ with} \]

\[V_{1q} = \exp \left\{ -2K_1^+ (c_q^+ c_q - \frac{1}{2}) \right\}\]. \tag{11.2-39}\]

For the case of odd \(n\) we still also need \(V_{1q}\) and \(V_{2q}\) for \(q = 0\) and \(q = \pi\). These operators are given by (see Problem 11.3):

\[V_{10} = \exp[-2K_1^+ (c_0^+ c_0 - \frac{1}{2})], \quad V_{1\pi} = \exp[-2K_1^+ (c_\pi^+ c_\pi - \frac{1}{2})], \]

\[V_{20} = \exp(2K_2^+ c_0 c_0), \quad V_{2\pi} = \exp(-2K_2^+ c_\pi c_\pi). \tag{11.2-40}\]

They are already in diagonal form and commute with each other.

To compute the partition function we need to evaluate – see Eq. (11.2-13) :

\[\mathcal{g}_{(even)} = \text{Tr}\left\{ \left[ (2 \sinh 2K_1)^{M/2} \prod_{q>0} V_q \right]^N \right\}, \quad V_q = (V_{1q})^{\frac{1}{2}} V_{2q} (V_{1q})^{\frac{1}{2}}, \tag{11.2-41}\]

where the \(q\)'s are chosen according to the upper line of (11.2-34). For a state with odd total occupancy the effect of the diagonal operators (11.2-40) must be added on to the above product; to keep their joint occupancy odd, we assume the 0-state is occupied while the \(\pi\)-state is left empty. Moreover, the wave-vectors of the lower line of (11.2-34) will be formally assigned the wave vector \(\vec{q}\). We thus have

\[\mathcal{g}_{(odd)} = \text{Tr}\left\{ \left[ (2 \sinh 2K_1)^{M/2} \prod_{\vec{q} \neq 0} V_{\vec{q}} \right] V_{0 \pi}^N \right\}, \quad V_{\vec{q}} = (V_{1\vec{q}})^{\frac{1}{2}} V_{2\vec{q}} (V_{1\vec{q}})^{\frac{1}{2}}, \quad V_{0 \pi} = V_{10 \pi} V_{20 \pi} \cdot \tag{11.2-42}\]
For now all wave-vectors will for convenience still be denoted by ‘$q$’. For a given $q$ the occupation-number space involves the four basic states

$$
\begin{align*}
|0_{-q}0_q\rangle, & \quad |0_{-q}1_q\rangle = c_q^\dagger |0_{-q}0_q\rangle, \\
|1_{-q}0_q\rangle = c_{-q}^\dagger |0_{-q}0_q\rangle, & \quad |1_{-q}1_q\rangle = c_{-q}^\dagger c_q^\dagger |0_{-q}0_q\rangle.
\end{align*}
$$

(11.2-43)

For simplicity we omitted in the kets all $q_i$ with $q'_i \neq q$ or $-q$; also, without loss of generality, we assumed that the total preceding occupancy is even so that there is no factor $(-1)^{\xi}$ to account for and we took $-q$ to precede $q$. [NB. The state $|0_{-q}0_q\rangle$ above is not the ground state, which will be defined later.] Since $V_{1a}$ only depends on the diagonal operators $n_q$ and $n_{-q}$, their action on $|0_{-q}1_q\rangle$ and $|1_{-q}0_q\rangle$ leaves these states unaltered except for a factor (c-number). As to $V_{2a}$, the cos part is diagonal and the sin part can produce off-diagonal matrix elements only when acting on states that differ by two in fermion occupancy. These states can therefore be deleted from the set (11.2-43), leaving as basis states the pseudo-spinor $\{ |1_{-q}1_q\rangle \ | 0_{-q}0_q\rangle \}$.\[14\]

Now we have $V_{1a}^{1/2} |1_{-q}1_q\rangle = [\exp((-K_q^i))] |1_{-q}1_q\rangle$ and $V_{1a}^{1/2} |0_{-q}0_q\rangle = [\exp(K_q^i)] |0_{-q}0_q\rangle$. Hence,

$$
V_{1a}^{1/2} = \begin{pmatrix}
\langle 1_{-q}1_q | V_{1a}^{1/2} | 1_{-q}1_q \rangle & \langle 1_{-q}1_q | V_{1a}^{1/2} | 0_{-q}0_q \rangle \\
\langle 0_{-q}0_q | V_{1a}^{1/2} | 1_{-q}1_q \rangle & \langle 0_{-q}0_q | V_{1a}^{1/2} | 0_{-q}0_q \rangle
\end{pmatrix} = \begin{pmatrix}
e^{K_q^i} & 0 \\
0 & e^{-K_q^i}\end{pmatrix}.
$$

(11.2-44)

To obtain the matrix for $V_{2a}$ we must employ more armour. Since both pseudo-spinor states involve two occupancies, Schultz et al. introduce pair operators $b_q = c_q c_{-q}^\dagger$ and $b_q^\dagger = c_q^\dagger c_{-q}$, similar to the Cooper-pair operators in BCS theory, discussed in the next chapter. Clearly, $c_q^\dagger c_{-q} + c_{-q}^\dagger c_q = b_q^\dagger + b_q$. For the diagonal operators note that, as far as operations on pair states are concerned, we can set $c_q^\dagger c_q = c_{-q}^\dagger c_{-q}^\dagger c_q = b_q^\dagger b_q$ and similarly for $c_{-q}^\dagger c_{-q}$. Now as indicated in connection with BCS theory [Eq. (12-6-2’)], the pair operators have commutation properties identical to spin lowering and raising operators, without the sign-change problem of the Jordan-Wigner transformation. Thus $b_q^\dagger \rightarrow \beta_q^z$, $b_q \rightarrow \beta_q^x$. In this (isomorphic) spin-space we have a representation by Pauli spin matrices such that

$$
\begin{align*}
c_q^\dagger c_q + c_{-q}^\dagger c_{-q} &= 2b_q^\dagger b_q \rightarrow 2\beta_q^+ \beta_q^- \equiv \beta_{q,z} + 1, \\
c_{-q}^\dagger c_q^\dagger + c_q c_{-q}^\dagger &= b_q^\dagger + b_q \rightarrow \beta_q^+ + \beta_q^- \equiv \beta_{q,x}.
\end{align*}
$$

(11.2-45a)

(11.2-45b)

We still need another operator

$$
\beta_q \equiv \beta_{q,z} \cos q + \beta_{q,x} \sin q,
$$

(11.2-46)

which has the important property that $(\beta_q)^2 = 1$ since $[\beta_{q,z}, \beta_{q,x}]_+ = 0$ [analogous to (11.1-13)]. For $V_{2a}$ this yields

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14 As elsewhere in this text, we use (...) for a row matrix and {…} for a column matrix to reduce space.
\[ c_{n+1} = M^{-1/2} e^{-i\pi/4} \sum_q e^{iqn} e^{iq} (\eta_q \cos \phi_q - \eta_{-q}^* \sin \phi_q) , \]
\[ c_{n+1}^\dagger = M^{-1/2} e^{i\pi/4} \sum_q e^{-iqn} e^{-iq} (\eta_q^* \cos \phi_q - \eta_{-q} \sin \phi_q) . \] (11.4-21)

For the annihilation operators we change the summation variables \( q' \to -q', q \to -q \); noting that \( \phi_{-q} = -\phi_q \), we get
\[ c_m^\dagger - c_m = M^{-1/2} e^{-i\pi/4} \sum_q e^{-iqm} (-\eta_{-q} + i\eta_q^*) (\cos \phi_q + i \sin \phi_q) , \]
\[ c_{n+1} + c_{n+1} = M^{-1/2} e^{-i\pi/4} \sum_q e^{-iqn} e^{-iq} (\eta_{-q} + i\eta_q^*) (\cos \phi_q - i \sin \phi_q) . \] (11.4-22)

Now \( (\eta_{-q} + i\eta_q^*) | \vec{0}_{-q} \theta_q \rangle = i | \vec{1}_{-q} \theta_q \rangle \). Next, acting on this with \( (-\eta_{-q} + i\eta_q^*) \) we see that \( -q' \) must be equal to \( q \), in order to recreate the ground state; the operator \( i\eta_q^* = i\eta_{-q} \) has no effect since it creates the state \( | \vec{1}_{-q} \theta_q \rangle \), which is orthogonal to \( | \vec{0}_{-q} \theta_q \rangle \). Therefore, the result is \( 26 \):
\[ \langle \Psi_0 | (c_m^\dagger - c_m) (c_{n+1}^\dagger + c_{n+1}) | \Psi_0 \rangle = M^{-1} e^{-i\pi/2} \sum_q e^{-iq(2n-m)} e^{-iq} e^{-i\pi/2} (\cos \phi_q - i \sin \phi_q)^2 . \]
\[ = M^{-1} \sum_q e^{-iq(n-m)} e^{-i(q+2\phi_q)} = a_{m,n} . \] (11.4-23)

Summing all pair products with their appropriate parity we obtain the Toeplitz determinant \( 27 \)
\[ \langle \sigma_{m+1} \sigma_{m+1} \rangle_{eq} = \begin{array}{cccc}
    a_{m,m} & a_{m,m+1} & \cdots & a_{m,m-1} \\
    a_{m+1,m} & \cdot & \cdot & \cdot \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{m-1,m} & \cdots & a_{m-1,m-1} & \cdot \\
\end{array} \rightarrow \begin{array}{cccc}
    a_0 & a_1 & \cdots & a_{m-1} \\
    a_{-1} & \cdot & \cdot & \cdot \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{m+1} & a_{m+2} & \cdots & a_0 \\
\end{array} . \] (11.4-24)

In the second form, used by MPW\(^8\), the correlation runs from the site \( m' \) to site \( 0 \), the rank of the determinant being \( m' \). The Toeplitz determinant, denoted as \( T(F) \), is closely related to the cyclic determinant \( C(F) \), as will now be indicated. \( 28 \)

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26 Since \( \phi_q \) is defined modulo \( \pi/2 \) [see (11.2-64)] we absorb the factor \( \exp(-i\pi) \) into \( \exp(-2i\phi_q) \).

27 In a Toeplitz determinant the element \( a_{ij} \) depends only on \( |i-j| \).

28 For a cyclic determinant \( a_{m+M,m'+M} = a_{m,m'} \); for a Toeplitz determinant of rank \( M \), \( a_{m,m} = 0 \) for \( m, m' > M \). For an example see Mattis, Op. Cit., p. 270.
Chapter XII

427, Eq. (12.3-4) has the creation and annihilation operators under the sum,

\[
\frac{1}{2V^2} \sum_{k,k',k''} a_k^+ a_{k''} a_{k'} \sum_q V_q \int \frac{d^3r}{v_{k+q}} e^{-i\mathbf{r} \cdot (\mathbf{k}-\mathbf{k}'-\mathbf{q})} \int \frac{d^3r'}{v_{k'+q'}} e^{-i\mathbf{r}' \cdot (\mathbf{k}'-\mathbf{k}''-\mathbf{q}')} \, ,
\]  
(12.3-4)

430, Eq. (12.3-18), 1st line: \( \mathcal{H}_{\text{grand}} = -\frac{1}{2} \varrho_0 \rho_0 n_0 l + \frac{1}{2} \sum_k (\mathbf{p}_k + \mathbf{p}_{-k} + 1)^{\frac{1}{2}} \left[ \varepsilon(\mathbf{k}) + \varrho_0 \rho_0 \right] \)

431, Eq. (12.3-24): remove \( \langle \rangle \) brackets, \( N = n_0 + \sum_k a_k^+ a_k = n_0 l + \sum_k a_k^+ a_k \).

444, §1, 17: \( \nu_0 \rho(0) \gg 1, \ldots \) similarly 449, §2, 6

447, §2, 3-7: we have rewritten the too terse outline in the book as follows:

\( \langle \rangle \) where \( |0\rangle \) is the vacuum state. Normalization gives \( C^{-1} = \prod_k (1 + g_k^2) \). For the normal state \( g_k = 0 \), |\( k| < k_F \) and \( g_k = 0 \), |\( k| > k_F \). Indeed,

\[
\prod_k c_k^+ |0\rangle = \prod_k c_{k,\sigma}^+ |0\rangle = \prod_k c_{k,\sigma} |0\rangle \equiv |O\rangle .
\]  
(12.6-6)

For the condensed state, the variational principle requires that upon variation of \( g_k \)

\[
W = \langle \Psi_0 | \mathcal{H}_{\text{gr.red.}} | \Psi_0 \rangle
\]  
(12.6-7)

be a minimum. Substituting (12.6-4) and (12.6-5), one finds that the vacuum state is reproduced iff \( W \) takes the form

\[
W = 2 \sum_k (\varepsilon(\mathbf{k}) - \mu) - \frac{g_k^2}{1 + g_k^2} + \sum_{k,k'} V_{kk'} \frac{g_k g_{k'}}{(1 + g_k^2)(1 + g_{k'}^2)} .
\]  
(12.6-8)

The BCS theory is most easily handled if the following quantities are introduced

\[
u_k = 1/\sqrt{1 + g_k^2}, \quad v_k = g_k / \sqrt{1 + g_k^2} .
\]  
(12.6-9)

with \( u_k^2 + v_k^2 = 1 \). This means that we must minimize the variational quantity \( W \):

\[
W = 2 \sum_k (\varepsilon(\mathbf{k}) - \mu) v_k^2 + \sum_{k,k'} V_{kk'} u_k v_k u_k v_{k'},
\]  
(12.6-10)

\[
\delta W = \delta \left\{ 2 \sum_k (\varepsilon(\mathbf{k}) - \mu) v_k^2 + \sum_{k,k'} V_{kk'} u_k v_k u_k v_{k'} \right\} = 0 .
\]  
(12.6-10')

When carrying out the variation we can eliminate \( \delta u \) since \( u \delta u + v \delta v = 0 \); hence,

457, §1,2ff. The clarity of these paragraphs is compromised by the over-usage of the term quasi-particles. We propose some minor changes by denoting the constituents of the quantum fluid as pseudo-particles; then – following Landau – the excitations above the ground state will be referred to as quasi-particles, whether massless or hole-particle pairs. An optional new version of p. 457-459 follows.
Due to the exclusion principle and its concomitant need for antisymmetrized wave functions, the theory of Fermi liquids is in many respects more complicated than the Bose-type systems we considered previously. There are a number of readable surveys from which the student may profit. We mention the older treatment of Pines and Nozières (1966\textsuperscript{47}), modern reviews by Baym and Pethick (1978\textsuperscript{48}) or Leggett (1975\textsuperscript{49}) and a useful section in Mahan’s book (2000\textsuperscript{50}). Detailed considerations need a diagrammatic treatment, as discussed in later sections. Here we just mention that the imaginary part of the self-energy, $\text{Im}\Sigma$, is found to be small, so that the spectral function $A(p,\omega)$ is peaked and resembles a delta function\textsuperscript{51}; we can thus for given $\hbar\omega$ associate a momentum $p$ with the excitations.

The constituents of the ground-state quantum liquid will in this text be denoted as *pseudo-particles*. Basically, we may think of these as ‘dressed’ $^3$He atoms, thereby acquiring an effective mass $m'$ (Leggett\textsuperscript{49}). Thus, a pseudo-particle in the Fermi-type spectrum can, in a sense, be regarded as an atom in the self-consistent field of the surrounding atoms. Experimental data suggest that $m' = 2.76m$, where $m$ is the bare mass of the atom. We can also surmise this as follows. The energy $\varepsilon_p$ should be zero for $p = 0$. A formal expansion about $p = 0$ will not have any odd terms – in contrast to a Bose liquid – because of particle-hole symmetry; the first important term is then $\varepsilon_p \propto p^2$, which defines the effective mass with $\varepsilon_p = p^2/2m'$. In accordance with Landau’s viewpoint we shall also introduce a self-consistent energy $\tilde{\varepsilon}_p$ which is more than $\varepsilon_p$; see the developments below.

Landau’s theory, as for the case of $^4$He, was guided by experimental results on the specific heat and the compressibility, required for the velocity of sound; it is therefore to be regarded as a phenomenological theory. The following basic assumptions will be made and are upheld by comparison of the theory with the experimental data. First of all, it must be assumed that the interactions can in some way be ‘adiabatically turned on’, so that the number of pseudo-particles remains the same as the number of separate atoms in the gaseous state. Secondly and as a consequence of this supposition, the pseudo-particles of the ground state obey Fermi–Dirac statistics. Fermi liquid theory now aims primarily at describing low-lying excited states, which naturally involve particle-hole pairs; these excitations or quasi-particles are fermions.\textsuperscript{52} In addition, there are two other types of excitations. Density fluctuations or zero-sound waves can be viewed as collective resonances of the

\textsuperscript{52} Following Landau, the denotations ‘quasi-particles’ and ‘excitations’ will be taken to be synonymous.
primary particle-hole fluid and are carried by phonons.\textsuperscript{52a} Further there are spin waves carried by paramagnons; they represent spin fluctuations involving pairs of opposite spin. They are inadequately accounted for by the original Fermi liquid theory, but a number of modern approaches, using quantum-field Hamiltonians of the types we studied before, have been developed. And, last but not least, we must mention that at very low temperatures pairing can take place, resulting in a boson-like superfluid. Contrary to Cooper pairs in BCS theory, these pairs usually have the triplet state, with $S = 1$.\textsuperscript{52b} Superfluidity in $^3$He was discovered in 1972 by Osheroff, Richardson and Lee.\textsuperscript{53} They identified two phases in the liquid, designated $A$ and $B$; a Nobel prize followed. Curiously, the theory for this phenomenon did exist already, since Balian and Werthamer\textsuperscript{54} developed the triplet pairing theory as an alternative for the BSC theory in 1963, in order to (possibly) explain some anomalous results in superconducting materials like Sn, Hg and others. All these developments will be briefly discussed in the next few sections.

We now return to Fermi liquid theory, which is our basic quest in this section. The ground state is a ‘quasi-vacuum’ – as for an electron gas – and will be designated as $|O\rangle$, its energy being $\varepsilon_0$. At $T=0$ the state is \textit{grosso modo} filled up to the Fermi radius $p_F$, although there is some fuzziness for the distribution in systems with interactions, cf. Ref. 51, loc. cit. Fig. 2.5; the distribution will be denoted by $n_0$. Mainly, this is a fictitious concept, except near the Fermi radius. The ground-state energy will be written as $\varepsilon_0 = \text{const} + \sum_{p,\sigma} n_0^0 p \varepsilon_p$, where $\sigma$ denotes the spin; the ‘constant’ needs no discussion. At non-zero temperatures the exclusion principle forces many excitations in states outside $p_F$; we shall denote their occupancy by $n_p$. We will assume that the average occupancy (Heaviside-like behaviour or step-down function) can still be represented by the ordinary F–D distribution,

$$ n_0^0 = n_F (\varepsilon_p - \varepsilon_0) = \lim_{T \to 0} \left\{ 1 / [ e^{\beta (\varepsilon_p - \varepsilon_0)} + 1 ] \right\}; $$

(12.7-1)

note that the Fermi-Dirac function will be denoted by the middle member for the present considerations. More important is the ‘difference distribution’ $\delta n_p = n_p - n_0^0$; for the energy with respect to the ground state we have,

$$ \delta = \varepsilon_0 + \sum_{p,\sigma} \varepsilon_p \delta n_p = \varepsilon_0 + \int \Delta^3_{p,\sigma} \varepsilon_p \delta n_p. $$

(12.7-2)

where $\Delta^3_{p,\sigma}$ is a shortcut notation for integration over $p$-space including the density of states $1/8\pi^3 \hbar^3$ [we set $V_0 = \text{unity}$] and summation over the spin $\sigma = \pm 1$.

It must now be born in mind that the total number of pseudo-particles in the

\textsuperscript{52a} The sound-wave phonons in $^3$He are the equivalent of plasmons in an electron gas, cf. Chapter XVI.

\textsuperscript{52b} It is customary to use the symbols $S$, $s$ and $\sigma$ (instead of $I$) for the nuclear spin.


ground state $N_0$ is not a constant of motion, nor is the number of quasi-particles (or excitations) $\Sigma \delta n_p$ above the ground state. Due to their fluctuations we need an ensemble with specified chemical potential, such as we used in previous sections. Fortunately, for the Fermi gas $\zeta$ is hardly affected by $\Sigma \delta n_p$ and we can use the ground state value. Instead of $\hat{E}$ we should consider the grand-ensemble Legendre transform

$$\hat{E} - \zeta N \equiv \hat{E}' , \quad N = N_0 + \Sigma \delta n_p ,$$

(12.7-3)

where we shall call $\hat{E}'$ the grand energy. Denoting further $\hat{E}_0 - \zeta N_0$ by $\hat{E}_0$, we find with trivial algebra,

$$\hat{E}' = \hat{E}_0 + \int \Delta^3 \rho(x)(\varepsilon(x) - \zeta) \delta n_p .$$

(12.7-4)

To be noted here is that $|p|$ is always very near the Fermi value $p_F$ so that $\varepsilon_p$ is close to $\zeta$; hence, the integrand is actually of second order smallness, i.e., $\sim (\delta n_p)^2$.

Landau now recognised that, after all, Eq. (12.7-4) cannot be the full story, since in the above each quasi-particle contributes independently to the grand energy. Therefore, he added a binary interaction term, which redefines the character of each excitation. Let $\varphi_{\rho\sigma, p'\sigma'}$ be a binary interaction energy; the grand energy up to (but not including) terms of $(\delta n_p)^3$ should then be modified from (12.7-4) to read:

$$\hat{E}' = \hat{E}_0 + \int \Delta^3 \rho(x)(\varepsilon(x) - \zeta) \delta n_p + \frac{1}{2} \int \int \Delta^3 \rho(x) \Delta^3 \rho(x') \varphi_{\rho\sigma, p'\sigma'} \delta n_p \delta n_p' \delta \sigma \delta \sigma' .$$

(12.7-5)

The interaction terms are spin-dependent, but they do not represent ordinary dipole-dipole coupling, which is negligibly small. Rather, the exchange hole around each quasi-particle causes an exchange energy which is spin-dependent analogous to the spin-spin coupling in a Heisenberg Hamiltonian. A full discussion is found in Leggett, Op. Cit. Generally, $\varphi_{\rho\sigma, p'\sigma'}$ contains a spin-symmetric and asymmetric contribution and we have

$$\varphi_{\rho\sigma, p'\sigma'} = \varphi_{\rho p}^{\sigma} + (\sigma \cdot \sigma') \varphi_{\rho p}^{\sigma'} ,$$

(12.7-6)

where $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ refers to the Pauli spin matrices. As to the two momenta of each term, we note that both have magnitudes which for all practical purposes are equal to $p_F$; therefore only their mutual angle $\gamma$ is relevant. Consequently, an expansion in Legendre polynomials is a natural representation. It is customary to remove a factor $\rho_F$, being the density of states at the Fermi surface, from the factors $\varphi$; we have $\rho_F = m^* p_F / \pi^2 h^3$. Thus we employ the expansions

$$\varphi_{\rho p}^{\sigma} (\gamma) = \frac{1}{\rho_F} \sum_{l=0}^\infty F_l^{\rho} P_l (\cos \gamma) .$$

(12.7-7)

We note that the coefficients $F_l^{\rho}$ and $F_l^{\sigma}$ are also called $F_l$ and $Z_l$ in many papers. Generally only the coefficients for $l = 0, 1$ are important. These, then, are the four
474, Eq. (12.9-3): altered as follows,

\[ U_f(t, t_0) \equiv U_f(t - t_0) \equiv U^{(0)}(t - t_0) U(t - t_0) = e^{iK\theta(t-t_0)}k e^{-(iK^3 + i\lambda t_0) (t-t_0)} k. \] (12.9-3)

§2, 1: We review some basic results, which have been further elaborated

480, §1, 8: Using the rules (12.9-36)

483, (iv), 2: from \( \tau_j \) to \( \tau_i \),

493, Fig. 12-24: Little contour \( C \) should be small \( c \).

497, Eq. (12.11-20), last part, \( \Psi^e_{\mu}(r', r^*) = \sum_k \Phi_k^e (r') \hat{\epsilon}_k^{\mu}(r') \),

499, §1, 1: \( K = H - \xi \sum_k n_k \)

506, §1, 2: Eqs. (12.11-5), (12.11-13) and (12.11-16).

508, Eq. (12.13-6), first line, \( \lim_{\delta \to 0} e^{i\delta} \)

513, §2, 7: Chapter XIII, Eqs. (13.4.11,12)

519, §1, 1-2: replace by: Although the numerator and the denominator are zero for \( q = 0 \), the ratio is finite, as we will see below. We write as always...

Eq. (12.13-49): remove the \( \beta^* \)s and in the second line write \( \int_0^\infty k dk n^0_k \).

§1, 4: where \( ^97a \)

footnote \( ^97a \) added:

\( ^97a \) The integral is rewritten as \( \varphi(x) = \frac{1}{\pi^2} \lim_{\delta \to 0} \int_0^\infty \xi d\xi e^{-\xi^2/4\delta} \ln \left[ 1 + \frac{2x}{\delta^2 - \xi^2} \right] \).

upon expansion of the logarithm we find, \( \varphi(0) = \frac{1}{\pi} \lim_{\delta \to 0} \int_0^\infty d\xi e^{-\xi^2} = 1 \).

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539, footnote \( ^9 \): See Reference 13 of Chapter VIII.

540, §2, 1: Let there be \( \hat{N} \Delta^3 r \) scatterers \( ^9a \)

footnote \( ^9a \) added:

\( ^9a \) E.g. \( \hat{N} \) is the density of ionized donors in electron-impurity scattering, cf. subsection 13.4.2 example (b) and Problem 13.5.

548, Fig. 13-5(a): the obtuse angle \( \theta \) should be labelled \( \Theta \).

548, Fig. 13-6(a): the angles \( \theta \) are as at right \( \to \)

551, §1, 7: \( \sum_k F(1 + \varepsilon_2 F). \to \hat{N} \Delta^3 r \).

557, §2, 11: Comparing with Eq. (1.3-8)

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567, §2, 2: \( u = \sum_i (1/\Delta^3 r) \Delta^3 k_i \varepsilon_{ki} f_i. \)

568, §2, 1: we set \( \Psi \) equal (cap \( \Psi \)
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591, Eqs.(15.1-12) and (15.1-14): the diffusion coefficient here is $D_n'$ (add the prime)
593, §1, 2 and 3: Section 15.3.
   §2, 12ff. Replace by new paragraph:
   At low temperatures at which the electron gas is fully degenerate, it is faster to
   use (15.2-7) as is; we can set $\partial f_n / \partial \epsilon = -\delta(\epsilon - \epsilon_F)$ and pull the relaxation time
   out of the integrand as $\tau_F$. For spherical energy surfaces, noting $v_kv_k \rightarrow \frac{1}{3} v_F^2 I$
   and $n(r) = n_F = \frac{k_F^3}{3\pi^2} [\text{cf. (8.4-11)}]$, one readily confirms the elementary result
594, §1, 12: this into (15.2-20) gives
595, Eq. (15.2-24): $S_n \cdot \nabla T$
600, Eq. (15.3-5): $J'_p(r,t) = -eJ'_p(r,t)$ (minus sign added)
   Eqs. (15.3-8) and (15.3-9): lhs, bold $r$; also, Eq. (15.3-10): $T(r,t)$
601, Eqs. (15.4-1) and (15.4-2): bold $r$ in $T(r)$
   §1, 14: $\gamma = (\xi - \epsilon_c) / k_B T$.
602, §1, 5: cap $\Psi$
612, Fig. 15-2a, 2b: the arrowheads on the vectors $k$ and $k'$ should be restored
617, §3, in Fig. 14-1 $\chi'$ be the angle of $v'$ with the polar axis (add the prime to chi)
619, §2, 6: where we choses the direction of $g_1$ such that the vector...
621, §3, 3: form as in (15.2-32).
622, §1, 1: This is substituted into (15.8-1) together with (15.7-10) and the terms
624, §1, 2: comparable to (15.6-15)
626, §2, 1 and 2: this should read
   Rests to evaluate the collision integral in first order (c.i.f.o.). From (15.10-3)
   and (15.9-1) we have, since $\overline{T_3}$ in the absence of a $B$-field was taken along $\alpha$,
627, Fig. 15-4: replace $\theta$ by $\Theta$
628, §2, 2: Thus from (15.10-16) and (15.9-6):
   §2, 7: … both results (15.10-18) and (15.10-20) are substituted into (15.9-5)
631, §1, 1: Previously we established [cf. (15.10-27)]
   §1, 3: With $A_p'$ given by (15.10-28), this yields
632, Problem 15.2: in Eq. (15.2-23).
   Problem 15.4, first line: was given in (15.2-20). Last line:
(b) From (15.2-20), obtain Landsberg’s result, (15.2-21).

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641, Eq. (6.2-14) and (6.2-15): we regret that the dotted symbols of mathtype are routinely too mall, in particular when occurring as subscripts; they have all been redone, e.g.,

\[
\langle \Delta \hat{B}(t) \rangle = \int_0^t dt' \phi_{\hat{B}}(t-t') F(t') ,
\]

(16.2-14)

\[
\phi_{\hat{B}}(t) = (1/\hbar) \text{Tr}[[A, \hat{B}(t)] \rho_{eq}] .
\]

(16.2-15)

666, Eqs. (16.6-12)-(16.6-15): there are factors two to be inserted,

\[
\chi(q, i\omega) = \lim_{\eta \to 0} \frac{1}{2\pi \hbar V_0} \int_{-\infty}^{\infty} d\omega'' \left\{ \frac{1}{\omega + \omega'' - i\eta} - \frac{1}{\omega - \omega'' - i\eta} \right\} . \tag{16.6-12}
\]

In the quantum limit (very large \(\omega\)) we find that \(\chi\) becomes real; combining the two parts in the curly brackets of (16.6-12), we find

\[
\chi(q, i\omega) = -\lim_{\eta \to 0} \frac{1}{\pi \eta \hbar V_0} \int_{-\infty}^{\infty} d\omega'' d\omega''' \text{Im} \chi(q, \omega''') / [(\omega - i\eta)^2 - \omega''^2]
\[
= \frac{2}{\pi} \lim_{\eta \to 0} \int_{-\infty}^{\infty} d\omega'' d\omega''' \text{Im} \chi(q, \omega''') / [(\omega - i\eta)^2 - \omega''^2] - \frac{2q^2 \langle n \rangle}{m \omega^2} , \tag{16.6-13}
\]

where we used (16.5-22) and the \(f\)-sum rule (16.5-35). Returning to the general form (16.6-12) and using (16.3-11), we find for the real and imaginary parts,

\[
\chi'(q, \omega) = -\frac{1}{\pi \hbar V_0} \int_{-\infty}^{\infty} d\omega'' S_d(q, \omega'') \frac{\omega''}{(\omega'' - \omega)^2} , \tag{16.6-14}
\]

\[
\chi''(q, \omega) = \frac{1}{2\hbar V_0} \{ S_d(q, -\omega) - S_d(q, \omega) \}. \tag{16.6-15}
\]

669, Eq. (16.6-33): \(d^3r\) replace by \(d^3k\)

\[\text{§2, 9 and 10: for the} k\text{-integral we note that the Heaviside function sets} \ k \leq k_F \text{ and we use} \langle n \rangle = k_F^3 / (3\pi^2), \text{cf. (8.4-11).}\]

674, \(\sigma_{\mu\nu} = \frac{\beta}{4} \lim_{T \to \infty} V_0 \int_T^T dt \sum_{k\ell} \text{Tr} \left\{ \rho_{\text{con}} \sum_{n_m} \langle k | j_{\nu} | k \rangle \langle \ell | j_{\mu} | \ell \rangle + \text{transp} \right\} . \tag{16.7-3}\]

\[\text{§2, 11: etc.; \text{transp} means the transpose,} \nu \leftrightarrow \mu.\]

675, \(\text{§1, 5: of the} \ \text{electron gas (omit ‘free’). 680, 15: complement space (the …)}\)

681, \(\text{§2, 7: which reads, writing} \ U^0_{\gamma}(t-t') = \exp[-i\gamma (t-t')/\hbar] \text{and noting that the} \ \text{perturbations} \ \lambda^\ell \ \text{are time-independent, last} \ \text{equation, second line:}\)

\[\times U_{\gamma}(t-t_n) \langle \gamma | [\gamma(t_n) | \gamma_{n-1} \rangle U^0_{\gamma_{n-1}}(t_{n-1} - t_{n-1}) \langle \gamma_{n-1} | [\gamma(t_{n-1}) | \gamma_{n-2} \rangle \ldots\]

\[\text{etc.}\]

\[\text{etc.}\]
682, Equation top line: \[ \times U^0 \left( \gamma_1 \left| t_2 - t_1 \right| \gamma_1 \right) U^0 \left( t_1 \right) . \] (16.9-15)

684, Eq. (16.9-32), second line: \[ \left| \left( \gamma^\nu \left| t^\beta \right| \gamma \right) \right|^2 \] (script \( \gamma \))

690, §1, 3 and 4: (even if not, this part goes with \( \lambda \) and does not give the diagonal singularity). Thus, …

697, Eq. (16.10-62) should be changed to read:

\[ \hat{A}_d^R \left( -i \hbar \tau \right) = e^{i \hbar \tau} \hat{A}_d^R \left( 0 \right) e^{-i \hbar \tau} = \hat{A}_d^R \left( 0 \right) \equiv \left( \hat{A}_d^R \right) . \] (16.10-62)

698, §1, 1: The dots have been inadvertently omitted: \( \hat{A}_d^R \left( 0 \right) \equiv \left( \hat{A}_d^R \right)_n = \hat{A}_d \),

701, §3, 5: (differentiation to the upper limit \( t \) of the integral gives a contribution of order \( \lambda \), which will be dismissed – as we did in the Heisenberg case). Hence,

703, Eq. (16.12-11), second line: \( (i / h)F(t)\partial^2[A, \bar{P}_{eq}] + (i / h)\partial^2 \partial_0 d't'F(t')... \)

715, Eq. (16.15-25) should read:

\[ \lim_{\hbar \to \infty} e^{i \hbar \xi} = \exp \left[ - \left( A_d \mp i \xi \right) \right] \] (16.15-25)

Problem 16.1, last line: show that there are no poles in the lower half plane.

716, Problem 16.4: Eq. (6) should read In coth(..).

Problem 16.5, part (a): Employing a convergence factor \( \exp(-\eta t) \), show…

717, Problem 16.7, Eq. (10): \[ \frac{2}{\pi} \int_0^\infty \text{Re} \sigma_{\nu \nu} (i \omega) d\omega = \frac{q^2 n}{m} \delta_{\nu \nu} , \] (10)

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722, Eq. (17.1-10), second line:

\[ \sum_{\eta^* \eta} \langle N_{\eta^*} \rangle_{eq} \left( 1 + \langle N_{\eta} \rangle_{eq} \right) Q(\xi^* \eta^*; \xi \eta) , \] (17.1-10)

725, Eq. (17.2-1), in the lhs, uses script symbol \( \bar{R} \); similarly in Eq. (17.2-2).

726, §2, 11: function \( \Psi_{\xi^* \xi} \).

728, Eq. (17.39), add: \( = 0 \).

729, Eq. (17.3-12)

first term = \[ \frac{1}{8 \pi^2} \frac{\partial}{\partial t} \int d^3 \Sigma \int d^3 u e^{i u \cdot \left( \Sigma - \eta \right)} \left[ h^3 \rho_1 (k, \Sigma, t) \right] = \frac{\partial f}{\partial t} , \] (17.3-12)

§2. 3: Wigner occupation function (16.10-56) and

730, §1, 2 and 3: Also we set \( \sum_{\xi \rightarrow \left( V_0 / 8 \pi^3 \right)} d^3 \kappa \). Then the last part of (17.3-10)

34 inside the curly brackets becomes:

739, Eq. (17.7-6): add \( (q / 2 V_0) \) in front of the sum sign

740, Eq. (17.7-8): add \( (\beta q / 2 V_0) \) in front of the sum sign

742, Page has been corrected and improved as follows:
boson bath states, we have by (17.1-15) and (17.1-16)
\[
\langle e^{-\lambda t} \rangle_b = \sum_{n_v} \langle \gamma | \sum_{k=0}^\infty (-M \xi t)^k \rangle_b / k!
\]
\[
= \sum_{n_v} |\langle \xi | \langle n_{\xi} \rangle | \sum_{k=0}^\infty (-\beta \xi t)^k / k! | \sum_{n_{\xi}} | \langle n_{\xi} \rangle | \langle n_{\xi} \rangle | e^{-\beta \xi t} .
\] (17.4-4)
which is formally correct, but of no value unless the operator on the rhs can be computed. Although $M$ is a linear operator, $\beta \xi$ is not. Therefore, we will limit ourselves to the case that $\beta \xi$ is linear or linearized and allows the definition of a relaxation time, i.e., $\beta \xi \to [1/\tau(\epsilon)]$. Substituting these results in (17.8-2) yields
\[
\langle 1 / i \omega + \Lambda_d \rangle_b = \sum_{n_{\xi}} |\langle n_{\xi} \rangle | \langle n_{\xi} \rangle | \frac{1}{i \omega + \tau^{-1}(\epsilon_{\xi})} .
\] (17.8-5)
The diagonal many-particle velocity operators are as usual given by
\[
\bar{v}_{\mu,d} = \sum_{\nu} \bar{n}_{\xi} \bar{\xi} | \nu_\mu \rightarrow \nu_\xi \rangle \cdot \bar{v}_{\nu,d} = \sum_{\xi} \bar{n}_{\xi} \bar{\xi} | \nu_\nu \rightarrow \nu_\xi \rangle .
\] (17.6-6)
Returning to (17.8-1) and going over to the grand-canonical ensemble, we find
\[
\sigma^{d}_{\mu \nu}(i \omega) = \frac{\beta^2}{V_0} \sum_{\xi, \xi'} \sum_{\nu} \text{Tr} \left\{ \rho_{\text{gean}} |\langle n_{\xi} \rangle | \langle n_{\xi} \rangle | \frac{\tau(\epsilon_{\xi})}{1 + i \omega \tau(\epsilon_{\xi})} \bar{n}_{\xi} \bar{\xi} \right\}
\times |\nu_\mu \rightarrow \nu_\xi \rangle (\xi_{\nu} \rightarrow \nu_{\xi}) .
\] (17.8-7)
In evaluating the trace we need the expression – with $X_{\xi}$ denoting the frequency term,
\[
\sum_{\xi, \xi'} \langle n_{\xi} \nu_{\xi} \rangle_{\text{eq}} (\xi_{\nu} \rightarrow \nu_{\xi}) (\xi_{\nu} \rightarrow \nu_{\xi}) X_{\xi} \rightarrow (i \omega)
\]
\[
= \sum_{\xi} \sum_{\xi' \neq \xi} \langle n_{\xi} \nu_{\xi} \rangle_{\text{eq}} \langle n_{\xi} \nu_{\xi} \rangle_{\text{eq}} (\xi_{\nu} \rightarrow \nu_{\xi}) (\xi_{\nu} \rightarrow \nu_{\xi}) X_{\xi} \rightarrow (i \omega)
\]
\[
+ \sum_{\xi} \langle n_{\xi} \nu_{\xi} \rangle_{\text{eq}} (\xi_{\nu} \rightarrow \nu_{\xi}) (\xi_{\nu} \rightarrow \nu_{\xi}) X_{\xi} \rightarrow (i \omega)
\]
\[
= \sum_{\xi} \langle n_{\xi} \nu_{\xi} \rangle_{\text{eq}} (\xi_{\nu} \rightarrow \nu_{\xi}) \sum_{\xi'} \langle n_{\xi'} \nu_{\xi'} \rangle_{\text{eq}} (\xi_{\nu} \rightarrow \nu_{\xi}) X_{\xi} \rightarrow (i \omega)
\]
\[
+ \sum_{\xi} \langle n_{\xi} \nu_{\xi} \rangle_{\text{eq}} (\xi_{\nu} \rightarrow \nu_{\xi}) \sum_{\xi'} \langle n_{\xi'} \nu_{\xi'} \rangle_{\text{eq}} (\xi_{\nu} \rightarrow \nu_{\xi}) X_{\xi} \rightarrow (i \omega) .
\] (17.8-8)
The next to last line of (17.8-8) is zero since the equilibrium current $\sum_{\nu_{\xi} \rightarrow \nu_{\xi}}$ vanishes. The last line involves the variance,
\[
\langle n_{\xi} \nu_{\xi} \rangle_{\text{eq}} - \langle n_{\xi} \nu_{\xi} \rangle_{\text{eq}}^2 = \langle \Delta n_{\xi} \nu_{\xi} \rangle_{\text{eq}} = \langle n_{\xi} \nu_{\xi} \rangle_{\text{eq}} (1 - \langle n_{\xi} \nu_{\xi} \rangle_{\text{eq}}) = -\beta^{-1} \frac{\partial \langle n_{\xi} \nu_{\xi} \rangle_{\text{eq}}}{\partial \epsilon_{\xi}} .
\] (17.8-9)
The final result now becomes
\[
\sigma^{d}_{\mu \nu}(i \omega) = -\frac{\beta^2}{V_0} \sum_{\xi} \frac{\partial \langle n_{\xi} \nu_{\xi} \rangle_{\text{eq}}}{\partial \epsilon_{\xi}} \frac{\tau(\epsilon_{\xi})}{1 + i \omega \tau(\epsilon_{\xi})} (\xi_{\nu} \rightarrow \nu_{\xi}) (\xi_{\nu} \rightarrow \nu_{\xi}) .
\] (17.8-10)
This result is exactly what we obtained earlier from the linearized QBE, cf. (17.6-6).
Chapter XVIII

791, Eq. (18.5-3):
\[ \frac{\partial}{\partial t} \left\langle e^{-s\alpha(t)} \mathbf{a} \right\rangle_{\mathbf{a}^\prime} = \left\langle e^{-s\alpha(t)} \left[ -\mathbf{s} \cdot \mathbf{A}(\mathbf{a}) + \frac{1}{2} \mathbf{s} \cdot \mathbf{B}(\mathbf{a}) + ... \right] \right\rangle_{\mathbf{a}^\prime}, \]
\[ + \left\langle e^{-s\alpha(t)} \left[ \mathbf{A}(\mathbf{a}) - \frac{1}{2} \mathbf{B}(\mathbf{a}) \cdot \mathbf{s} - \frac{1}{2} \mathbf{s} \cdot \mathbf{B}(\mathbf{a}) + ... \right] \right\rangle_{\mathbf{a}^\prime}, \]
\[ + \left\langle e^{-s\alpha(t)} \left[ \mathbf{B}(\mathbf{a}) + ... \right] \right\rangle_{\mathbf{a}^\prime}. \] (18.5-3)

801, Fig. 18-3(a): trap level should be close to the valance band

803, Eq. (18.5 - 72): \( i_0 = -\frac{n_0 (\kappa + \delta)}{2\kappa} + \frac{1}{2\kappa} [(\kappa + \delta)^2 n_0^2 + 4\delta \kappa n_0 I]^{1/2}. \) (18.7-2)

807, §1, lines 1-3 should be changed as follows,

Averaging over an ensemble with initial values fixed to \( \mathbf{a}(0) = \mathbf{a}^\prime \), we have \( \langle \mathbf{a}(\Delta t) - \mathbf{a}^\prime \rangle = -M \langle \Delta \mathbf{a} \rangle_{\mathbf{a}^\prime} \Delta t. \) Now, using the definition of the first-order F–P moment (18.5-17) and dividing by \( \Delta t \), we also find \( \mathbf{A}(\mathbf{a}^\prime) = -M \langle \Delta \mathbf{a} \rangle_{\mathbf{a}^\prime} \), in accord with (18.5-4). Next, we multiply (18.6-8) by its transpose and average again with \( \mathbf{a}(0) = \mathbf{a}^\prime \); thus, \( \langle \mathbf{a}(\Delta t) - \mathbf{a}^\prime \rangle (\mathbf{a}(\Delta t) - \mathbf{a}^\prime) \rangle_{\mathbf{a}^\prime} = ... \)

811, Eq. (18.8-5): \( \Pi e^{i k_0 \cdot z - B k^2 t}. \)

815, Eq. (18.7-31), middle line needs a + sign in the last bracket:
\[ = \int \left[ \nabla_x \mathbf{f} \cdot d\mathbf{v} - \frac{1}{\beta} \left( \nabla_x \mathbf{f} \right) \cdot \frac{d\mathbf{v}}{d\mathbf{r}} \right] = \int \left[ \frac{\partial \mathbf{f}}{\partial \mathbf{v}} \cdot d\mathbf{v} + \frac{\partial \mathbf{f}}{\partial \mathbf{r}} \cdot d\mathbf{r} \right]. \]
816, Eq. (18.7-32), second line:
\[ \frac{k_B T}{m \beta} \nabla_{r'} \cdot \left\{ \nabla_{r'} P \left( r'- \beta^{-1} v, v, t | r_0, v_0 \right) - \frac{m K}{k_B T} P \left( r'- \beta^{-1} v, v, t | r_0, v_0 \right) \right\} \, dv. \]

816 Eq. (18.8-33):
\[ \frac{\partial}{\partial t} P(r', t | r_0, v_0) = \frac{k_B T}{m \beta} \nabla_{r'} \cdot \left\{ \nabla_{r'} P(r', t | r_0, v_0) + \frac{1}{k_B T} (\nabla_{r'} \beta) P(r', t | r_0, v_0) \right\}, \]

817, Eq. (18.7-39): \[ \text{[ ] of denominator last exponent: } \left[ 1 - e^{-2(\omega^2/\beta)} \right] \]

819, Eq. (18.9-11), first line,
\[ D(i\omega) = \frac{1}{2\pi} \int_0^\infty dt \int_0^\infty d\omega \hat{F}(\omega') e^{-i(\omega-\omega')t} \]

820, Eq. (18.9-15):
\[ \mu(i\omega) = \eta \left\langle \frac{\Delta N^2}{\langle N \rangle} \right\rangle \int_0^\infty dt e^{-i\omega t} \int_0^\beta d\tau \langle \Delta v_j \rangle \langle -i\hbar \tau \Delta v_j(t) \rangle. \]

820, §1, 6: result (15.2-17)!

822, Eq. (18.9-2b): replace \( S_{a a} \) by \( \mathbb{S}_{a a} \)

822-824. On these pages the subscripts “\( a a \)” on \( \Phi, S \) and \( \Psi \) are confusing, since these quantities are already tensors; remove in Eqs. (18.9-1a), (18.9-1b), (18.9-2a), (19.9-2b), (18.10-2), (18.10-5)-(18.10-10).

826, Eq. (18.10-25):
\[ \overline{M}_\theta = \sum_M W(M, \theta) = \left\{ \lambda^{t+\theta} N(t') dt' \right\} = \lambda \theta \langle N \rangle, \quad (18.10-25) \]

827, §2, 4: Let the light intensity on a photocathode of unit efficiency be …
Eq. (18.10-34), lhs: \( W(M, \mathcal{J}) = \quad \)
§2, 11 and 12: (18-10-24). For the counting variance in \( (t, t + \mathcal{J}), \langle \Delta M_{\mathcal{J}} \rangle \), we obtain a result analogous to (18.10-27). With the usual change of variables and integrating separately over the two regions of Fig 18-9, we then obtain

828, §1, 1: Let now \( \bar{m} \) be the mean photon rate and let…

832, §2, 8: the Fourier transform is \( e \) and the …

834, §3, 4: For even analytic spectra we extend the frequency in the complex plane, \( \omega \rightarrow w \), thus
\[ F(s) = \frac{3}{\pi} \int_C dw \frac{w^2}{s(s^2 + 4w^2)(s^2 + w^2)} S_\gamma(w), \quad (18.12-11) \]

835, §2, 1-3. replace \( \omega's \) by \( w's \); poles occur at \( w = is, is / 2 \), etc.

838, Eq. (18.12-22), last line: denominator should be \( \omega^{n-2} \)

840, Eq. (18.12-30): – sign in front of the rhs

843, Eq. (18.5-2), first line:
\[ S_\alpha(\omega) = 2 \omega^2 \text{Re} \left[ -i \omega (M + i \omega) (M + i \omega - M) M^{-1} B \right] \]

845, Eq. (18.15-12), first line: \[ p_{12} = \beta np, \quad p_{21} = \alpha = \beta n_i^2; \]

845, 3, 6: Apart from the entry for \( p_{21} \), we made…

846, Fig. 18-14: in caption: interchange \( p_{12} \) and \( p_{21} \); also, \( p_{31} = \delta n_i \)
848, Fig. 18-15(a): draw a horizontal line at ‘1’ on the ordinate; this needs the designation ‘thermal noise’.

850, §2, 16: add ‘sub’ to the $\delta$’s.

854, #18.5: In the last entry of the $M$-matrix, remove the $/\rho$.

855, #18.8, replace $\tau$ by $t$:

$$
\begin{align*}
\sum_{m=0}^{\infty} \frac{(-n)!}{m!(n-m)!} (-q)^m &= 1 + \sum_{m=1}^{\infty} \frac{(-n)(-n-1)...(-n-m+1)}{m!} (-q)^m = (1-q)^{-n}.
\end{align*}
$$

856, #18.14, part (c): replace $\omega \rightarrow w$ in the complex plane:

$$
\langle u(t) u(t+\theta) \rangle = \frac{k_B T}{2\pi m} \int_{\infty-i\epsilon}^{\infty+i\epsilon} \frac{e^{iw\theta}}{iw+\gamma(iw)} dw,
$$

856, last line part (c): Assuming further that $\lim_{|w|\rightarrow\infty} \gamma(iw) = \text{finite}$, show…

Chapter XIX

858, first equation, in lhs: $P(X_k)$


Eq. (19.1-18): The products run from $k=0$ to $N$

860, (iii), 1: Bayes’ rule (19.1-3).

862, Eq. (19.2-11): the sum runs from $\ell=0$ to $\infty$

863, §1, 2: Hence, we have the boson distribution

after Eq. (19.2-17’) insert:

The binomial theorem now takes the form of the well-known expansion

$$
\sum_{m=0}^{\infty} \binom{-n}{m} (-q)^m = 1 + \sum_{m=1}^{\infty} \frac{(-n)(-n-1)...(-n-m+1)}{m!} (-q)^m = (1-q)^{-n}.
$$

For the generating function of this distribution, employing (19.2-12), we find

$$
G(z) = \sum_{m=0}^{\infty} z^m P(m|n) = [1 + M (1-z)]^{-n}.
$$

863, footnote has been changed to

The negative factorials in these formulae are singular [cf. Problem 4.14], but their ratio is finite.

869, §2.5: $e^{E_i} \ell_i \sim e_G$, where …

872, Eq. (19.5-15): put an equal sign = after $\phi_{N,0}$

880, §2, 4: by $\tilde{G}(r,r',i\omega) = \tilde{G}(r,r',-i\omega)$ [add tildes]

§2, 5: $a(r,t)$ replace by $\tilde{a}(r,\omega)$
889, Fig. 19-8(b): lettering insert II: \( \tau = \tau_a \)

892, §1, 13: The noise spectra are found from (19.6-26), first line.
in last line of bottom formula: replace \( p_s (r) \) by \( p_s (r') \)

893, Eq. (19.8-17): replace \( p_s (r) \) by \( p_s (r') \)

894, §2, 3: extension of (19.8-3)

Chapter XX

906, Eqs. (20.3-5) and (20.3-6):
\[
\mathcal{E}^+(r,t)|\{\alpha_q\} = \sum_q i(\hbar \omega_q / 2V_0)^{1/2} \alpha_q u^*_q(r)e^{-i\omega_q t} |\{\alpha_q\}\rangle \equiv \mathcal{E}^+(r,t)|\{\alpha_q\}\rangle. \tag{20.3-5}
\]
Accordingly, the ‘pseudo-classical field’ takes the form
\[
\mathcal{E}^+(r,t) = \sum_q i(\hbar \omega_q / 2V_0)^{1/2} \alpha_q u^*_q(r)e^{-i\omega_q t}. \tag{20.3-6}
\]

915, §2, 3: change names to Hanbury–Brown and Twiss.

Appendix A

923, first line. Section A.1 should have been named: The Schrödinger Picture

926, Eq. (A.3-3) should read: \( dU^0(t,t_0) / dt = (1 / \hbar i) \hat{H}^0U^0(t,t_0) \).

927, Eq. (A.3-9) should be improved in clarity:
\[
\langle \gamma | \tilde{U}^{(n)}(t_0, t_0) | \gamma_0 \rangle = \left( \frac{i}{\hbar} \right)^n \sum_{\gamma_{n-1}} \int_{t_0}^t dt_n \int_{t_0}^{t_n} dt_{n-1} \cdots \int_{t_0}^{t_2} dt_1 
\times e^{-i\delta_0 (t-t_0)/\hbar} \langle \gamma | \tilde{U}^{(n)}(t_n) | \gamma_{n-1} \rangle e^{-i\delta_{n-1} \omega (t_{n-1})/\hbar} \langle \gamma_{n-1} | \tilde{U}^{(2)}(t_{n-1}) | \gamma_{n-2} \rangle \cdots
\times e^{-i\delta_{n-1} \omega (t_1)/\hbar} \langle \gamma_1 | \tilde{U}^{(2)}(t_1) | \gamma_0 \rangle e^{-i\delta_{n-1} \omega (t_0)/\hbar}. \tag{A.3-9}
\]

928, Eq. (A.3-12) should read: \( a_{k,H}^\dagger (t) = U_{k,H}^\dagger (t) U_{j}^\dagger (t), \quad a_{k,H} (t) = U_{k,H} (t) U_{j} (t), \quad a_{k,i}^\dagger (t) = U_{k,i}^\dagger (t) U_{j}^\dagger (t), \quad a_{k,i} (t) = U_{k,i} (t) U_{j} (t). \)

Appendix B

935, and following pages. The dots on the symbols are too small and the dotted indices are barely discernible. All formulas with such symbols have been redone and will as such appear in a second, revised printing.