Electromagnetic angular momentum and quantum mechanics

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(Received 22 September 1997; accepted 27 December 1997)

A quick way of arriving at the Dirac quantization condition between electric and magnetic charges is to require that the electromagnetic field angular momentum of a Thomson dipole (a magnetic monopole and an electric charge) equal some integer multiple of the fundamental unit of quantum mechanical angular momentum, \( \hbar /2 \). Applying this same type of argument to the electromagnetic field angular momentum carried by a magnetic dipole–electric charge system leads to an infinite number of different quantization conditions, and an apparent incompatibility between quantum mechanics and the dipole–charge system. However, a more careful analysis shows that the particle plus field angular momentum of this system does satisfy the standard angular momentum commutation relationships and is therefore a good quantum mechanical angular momentum. This emphasizes that caution must be taken when applying such semiclassical quantization arguments. Finally, a possible connection between this dipole–charge field angular momentum and the nucleon spin crisis is given. © 1998 American Association of Physics Teachers.

I. INTRODUCTION

One interesting aspect of including magnetic charges in electromagnetism is that the system of a point electric charge \( +e \) and a point magnetic charge, \( +g \), carry an angular momentum in their electric and magnetic fields. By integrating this field angular momentum density over all space the total field angular momentum is

\[
\mathbf{L}_{\text{em}} = \frac{eg}{c} \hat{r},
\]

where \( \hat{r} \) is a unit vector which points from \( +e \) to \( +g \). This result was used by several authors to arrive at the Dirac quantization condition between electric and magnetic charge. Requiring that the field angular momentum in Eq. (1) come in integer multiple of \( \hbar /2 \) immediately leads to the Dirac quantization condition of \( eg/c=n(\hbar/2) \). This semiclassical argument can be found in a number of texts and it has the virtue that one does not need to discuss Dirac strings. It only uses the fact that the electric and magnetic fields of the particles are of Coulombic form. A crucial point to this argument is that the field angular momentum in Eq. (1) depends on the charges, \( e \) and \( g \), but not on the distance between them. In this paper we will examine the less frequently discussed, but more physically realistic, system of an electric charge moving in the field of a magnetic dipole. This system also carries a field angular momentum. Applying the semiclassical quantization argument to the dipole–charge case leads to an apparent conflict with quantum mechanics. This indicates that the success of this argument for the Thomson dipole is somewhat of an “accident,” and that caution should be taken when applying this argument. By examining the dipole–charge system more carefully it is found that the particle plus field angular momentum is a proper quantum mechanical angular momentum. That a dipole–charge system carries a field angular momentum is remarked upon only occasionally, as in Feynman’s lectures. This field angular momentum is usually not taken into account, although many real systems such as the hydrogen atom have particles with both charge and magnetic dipoles. By making some rough order of magnitude calculations we will argue that for atomic size systems this angular momentum probably does not play any kind of significant role. For nuclear size systems it may be important, and could play a part in explaining where the nucleons get some of their internal angular momentum, especially if a quantum chromodynamics (QCD) version of this field angular momentum is considered.

II. CLASSICAL FIELD ANGULAR MOMENTUM

First we will find the classical field angular momentum for the dipole–charge system. To do this we should integrate the field angular momentum density over all space

\[
\mathbf{L}_{\text{em}} = \frac{1}{4\pi c} \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) d^3x,
\]

where \( \mathbf{E} = e\mathbf{r}/r^3 \) is the Coulombic electric field of the charged particle, \( +e \), located at \( \mathbf{R} \) (so \( \mathbf{r}' = \mathbf{r} - \mathbf{R} \)), and \( \mathbf{B} = (3\mathbf{r}(\mathbf{r} \cdot \mathbf{M})/r^5) - (\mathbf{M}/r^3) \) is the magnetic field of the dipole,
M, located at the origin. Instead of doing the integral in Eq. (2) to find \( \mathbf{L}_{\text{em}} \) directly we will use the result of Eq. (1) for the field angular momentum from a Thomson dipole. Taking two magnetic charges—one with charge \(+ g\) at the origin and another with charge \(- g\) at \( \mathbf{P} \)—placing an electric charge \(+ e\) at \( \mathbf{R} \), and adding up the field angular momentum of each magnetic charge–electric charge set gives

\[
\mathbf{L}_{\text{em}} = \frac{e}{c} \left( -\frac{g \mathbf{R}}{\mathbf{R}} + \frac{g(\mathbf{R} - \mathbf{P})}{\mathbf{R} - \mathbf{P}} \right). \tag{3}
\]

Expanding \( 1/|\mathbf{R} - \mathbf{P}| = (1/R) + (\mathbf{R}/\mathbf{P}/R^3) + \cdots \) and taking the limit \( \mathbf{P} \to 0, \ g \to \infty, \) and \( \mathbf{P} \to -\mathbf{M} \) gives

\[
\mathbf{L}_{\text{em}} = \frac{e\mathbf{M} \cdot \mathbf{E}}{c^{-2}} - \frac{e\mathbf{R} \cdot \mathbf{M}}{c^{-2}} \tag{4}
\]

for the field angular momentum in the dipole–charge system. One subtle point in this is that the dipole formed by taking the above limit of two magnetic point charges is different near the origin from a magnetic dipole made from a current. Physically this comes about since a magnetic dipole produced by a current loop, which has its magnetic moment pointing along the \( z \) axis, will have a magnetic field which near the center of the loop also points along the \( z \) axis. For a magnetic dipole made from two point magnetic charges, the magnetic field near the center between the two monopoles will point along the negative \( z \) axis. Mathematically this is taken into account by adding a delta function term to the standard magnetic dipole field. One adds \( + 8\pi \mathbf{M} \delta(r)/3 \) for the magnetic dipole produced by currents and \( - 4\pi \mathbf{M} \delta(r)/3 \) for the magnetic dipole produced by monopoles. The limiting procedure for obtaining Eq. (4) only produces the \( (3\mathbf{r} \cdot \mathbf{M})/r^3 - (\mathbf{M}/r^3) \) part of the magnetic dipole field, and not the delta function part. Thus for each of the two cases one should add to Eq. (4) the contribution of this delta function part of the magnetic dipole field. In both cases, however, the angular momentum density coming from the delta function part of the magnetic field will be of the form

\[
\mathbf{r} \times (\mathbf{E} \times \mathbf{M} \delta(r)). \tag{5}
\]

When this is integrated over all space it will vanish because of the delta function and the factor of \( \mathbf{r} \) in the integrand. Thus Eq. (4) is the expression for the field angular momentum for either a magnetic dipole produced by current or by monopoles.

It can be shown explicitly that the field angular momentum is required in order that the total angular momentum of the dipole–charge system be conserved. To have the angular momentum of the system be conserved the net torque on the system should be zero. For the charged particle moving in the field of the dipole the torque is

\[
\tau_{\text{charge}} = \frac{e}{c} (\mathbf{r} \times (\mathbf{V} \times \mathbf{B})) = \frac{e}{c} (\mathbf{V} \times (\mathbf{R} - \mathbf{B}) - \mathbf{B} \times (\mathbf{V} \times \mathbf{B})) = \frac{e}{c} \left( \frac{2\mathbf{V} \cdot (\mathbf{V} \times \mathbf{B})}{c} - \frac{3\mathbf{R} \cdot (\mathbf{V} \times \mathbf{B})}{c} + \frac{\mathbf{R} \cdot (\mathbf{V} \times \mathbf{B})}{c} \right), \tag{6}
\]

where \( \mathbf{V} \) is the velocity of the charged particle, and the form of the magnetic dipole field, at the location, \( \mathbf{R} \), of the charged particle has been used. The torque associated with the field angular momentum is [using Eq. (4)]

\[
\tau_{\text{em}} = \frac{d\mathbf{L}_{\text{em}}}{dt} = \frac{e}{c} \left( -\frac{\mathbf{M} \cdot (\mathbf{R} \times \mathbf{V})}{R^3} - \frac{\mathbf{V} \cdot (\mathbf{M} \times \mathbf{R})}{R^3} + \frac{3\mathbf{R} \cdot (\mathbf{M} \times \mathbf{V})}{R^3} \right). \tag{7}
\]

Finally, the moving charge creates a magnetic field at the location of the magnetic dipole which puts a torque on it. The magnetic field created by the moving charge is \( \mathbf{B} = e(\mathbf{V} \times (-\mathbf{R}))/cR^3 \). The minus sign occurs because the magnetic field of the moving charge is evaluated at the location of the dipole. This creates a torque on the dipole of

\[
\tau_{\text{dipole}} = \mathbf{M} \times \mathbf{B} = -\frac{e}{c} \left( \frac{\mathbf{M} \cdot (\mathbf{V} \times \mathbf{V})}{R^3} - \frac{\mathbf{V} \cdot (\mathbf{M} \times \mathbf{R})}{R^3} \right). \tag{8}
\]

Summing up the three torques from Eqs. (6)–(8) gives

\[
\tau_{\text{total}} = \tau_{\text{charge}} + \tau_{\text{em}} + \tau_{\text{dipole}} = 0. \tag{9}
\]

Therefore the total angular momentum for this system (charge+dipole) is conserved as is expected for a closed system. This shows that the field angular momentum must be taken into account if the angular momentum of this system is to be conserved.

### III. Quantum Mechanical Angular Momentum

For the Thomson dipole, several authors noticed\(^4\) that by requiring the electromagnetic field angular momentum to come in some integer multiple of the quantum mechanical unit of angular momentum (i.e., \( \hbar/2 \)), the Dirac quantization condition was obtained in a quick and simple manner. The main aim of this paper is to show that this semiclassical argument must be used with caution, since for the magnetic dipole–electric charge case it leads to apparent problems with quantum mechanics. Taking the dipole \( \mathbf{M} \) to be aligned along the \( z \) axis, we find from Eq. (4) that the magnitude of the field angular momentum along the \( z \) axis is

\[
|L_{z}| = \frac{eM(1 - \cos^{2} \theta)}{cR}, \tag{10}
\]

where \( \theta \) is the standard angle from the \( z \) axis. If the \( z \) component of the field angular momentum is required to equal some integer multiple of \( \hbar/2 \) a problem is encountered. If the electric charge \( e \) is placed at a given position (so that \( R \) and \( \theta \) are fixed), then one gets a certain condition between \( e \) and \( M \) by setting Eq. (10) equal to \( n\hbar/2 \). If the electric charge is at some neighboring position, with slightly different \( R \) and \( \theta \), one gets a different quantization condition, which will usually be incompatible with the first quantization condition. For every possible position of the electric charge one will get a different quantization condition, which will be incompatible with the quantization conditions from other positions. (Some positions will yield the same quantization condition—for example, \( \theta = \pi/4 \) and \( \theta = 3\pi/4 \) for the same \( R \) will yield the same condition. But in general different positions will yield different conditions). Unlike the case of the Thomson dipole,
where the field angular momentum is independent of the distance between the constituents, in the dipole–charge case the field angular momentum depends on the distance between the constituents. If the parts of the dipole–charge system could only take up certain discrete positions relative to one another, in terms of $R$ and $\theta$, then this semiclassical method would work, but such a positional quantization requirement is untenable for the dipole–charge system.

Applying the semiclassical quantization method to the dipole–charge system seems to lead to the strange conclusion that this system is incompatible with quantum mechanical angular momentum, despite the fact that there are a number of real systems which possess both charges and magnetic dipoles, and which are correctly treated using quantum mechanics. We want to show that when this dipole–charge field angular momentum is correctly considered, it does behave as a good quantum mechanical angular momentum. We will take the commutation relationship

$$[L_i, L_j] = i\hbar\epsilon_{ijk}L_k,$$

as the main property of a proper quantum mechanical angular momentum. The angular momentum of the charged particle plus the field angular momentum is given by

$$L_i = \epsilon_{ilm}R_iD_m + \frac{eM_i}{cR} - \frac{eR_kmM_k}{cR^3},$$

where the last two terms are the field angular momentum and the first term is the relative orbital angular momentum term, but with the ordinary momentum operator replaced by the covariant momentum, $p_m \rightarrow D_m = -i\hbar\partial_m - eA_m/c$, with $A_m$ being the vector potential. By showing that the angular momentum in Eq. (12) obeys the commutation relationship in Eq. (11) we will demonstrate that the angular momentum in Eq. (12)—and also the total angular momentum which adds the spin of the dipole to Eq. (12)—is a proper quantum mechanical angular momentum. Inserting this into the commutator in Eq. (11) gives

$$[L_i, L_j] = \epsilon_{ilm}\epsilon_{pqj}[R_iD_m, R_pD_q] + \epsilon_{ilm}R_i\left[D_m, \frac{eM_j}{cR} - \frac{eR_kmM_h}{cR^3}\right] - \epsilon_{jpq}R_p\left[D_q, \frac{eM_i}{cR} - \frac{eR_kmM_h}{cR^3}\right].$$

The first term in the first line in Eq. (13) can be written out as

$$[R_iD_m, R_pD_q] = R_iR_p[D_m, D_q] + R_i[D_m, R_p]D_q + R_p(R_iD_m)D_q$$

$$= i\frac{e\hbar}{c} \epsilon_{mkq}R_iR_pB_k - i\hbar\delta_{mp}R_iD_q$$

$$+ i\hbar\delta_{iq}R_pD_m,$$

where the relationships $[D_m, R_p] = -i\hbar\delta_{mp}$ and $[D_m, D_q] = i(\epsilon\hbar/c)\epsilon_{mkq}B_k$ (Ref. 9) have been used ($B_k$ = $k$th component of the total magnetic field). Putting the $\epsilon_{ilm}\epsilon_{pqj}$ factor back in the expression from Eq. (14), and using the identity

$$\epsilon_{jpq}\epsilon_{mkq} = \delta_{jm}\delta_{pk} - \delta_{jk}\delta_{pm},$$

leads to the following final form for this term:

$$i\hbar\left(\frac{e}{c}\epsilon_{ijk}R_iR_pB_p + eR_jD_j - R_iD_i\right)$$

$$= i\hbar\epsilon_{ijk}\left(\epsilon_{klm}R_mD_m + \frac{e}{c}R_iR_pB_p\right),$$

(16)

where the antisymmetry property of $\epsilon_{ilm}$ was used, Eq. (15) was used, and some dummy indices were renamed. The second term in Eq. (13) can be written out as

$$\epsilon_{ilm}R_i\left(\frac{eM_j}{c}D_m + \frac{1}{R}\right) - \frac{eM_h}{c}D_m, \frac{R_hR_h}{R^3}\right)$$

$$= i\frac{e\hbar}{c}\epsilon_{ilm}\left[R_p\frac{R_mR_m}{R^3} + \frac{R_hM_h\delta_{mj}}{R^3}\right],$$

(17)

where the formula $[D_m, f(R)] = -i\hbar\partial_m(f(R))$ has been used several times. The third term in Eq. (13) can be obtained from Eq. (17) by letting $i \rightarrow j$. Defining the vectors $a_i = \epsilon_{ilm}R_m$ and $b_j = R_j/R^3$, one can write the sum of the second and third terms in Eq. (13) as

$$i\frac{e\hbar}{c}(a_p-b_j-b_j) - 2i\frac{e\hbar}{c}\epsilon_{ijk}R_k\frac{M_pR_h}{R^3}$$

$$= i\frac{e\hbar}{c}\epsilon_{ijk}\epsilon_{klm}b_m - 2i\frac{e\hbar}{c}\epsilon_{ijk}R_k\frac{M_pR_h}{R^3}$$

$$= i\frac{e\hbar}{c}\epsilon_{ijk}\left[M_k\frac{3R_kM_pR_j}{R^3}\right],$$

(18)

where Eq. (15) has been used several times and some dummy indices changed. Combining this with the first term from Eq. (13) gives

$$[L_i, L_j] = i\hbar\epsilon_{ijk}\left(\epsilon_{klm}R_mD_m + \frac{e}{c}R_iR_pB_p\right)$$

$$+ i\frac{e\hbar}{c}\epsilon_{ijk}\left[M_k\frac{3R_kM_pR_j}{R^3}\right]$$

$$= i\hbar\epsilon_{ijk}\left(\epsilon_{klm}R_mD_m + \frac{e}{c}R_iR_pB_p\right),$$

(19)

where we have inserted the magnetic dipole field [i.e., $B_j = (3R_kM_p)/(R^3) - (M_p/R^3)$] in going from the first to second line and $L_k$ is the angular momentum given in Eq. (12). The charged particle’s angular momentum plus the field angular momentum thus obey the correct commutation relationships to be considered a quantum mechanical angular momentum. The total angular momentum for this system, which would include the spin of the central dipole, also obeys the angular momentum commutation relationships. By adding the spin of the dipole to the particle plus field angular momentum one immediately finds, in the usual way, that this total system angular momentum satisfies the commutator in Eq. (11). It should be noted that the electromagnetic field angular momentum given in Eq. (4) is not by itself a proper
quantum mechanical angular momentum, since it does not obey the commutator in Eq. (11). It is only the combination in Eq. (12) which satisfies the conditions for being a quantum mechanical angular momentum.

IV. PHYSICAL SYSTEMS WITH FIELD ANGULAR MOMENTUM

After having shown that the field angular momentum of a dipole–charge system does obey the usual commutation rules when considered correctly, one might ask why this field angular momentum is not considered when discussing physical systems which have both charges and magnetic dipoles. As a first example we will consider the hydrogen atom in its ground state. The proton and electron each have a charge and a magnetic dipole. This system is more complicated than the electric charge–magnetic dipole system considered in the previous section. There are now two contributions to the field angular momentum: first from the interaction of the proton’s charge with the intrinsic magnetic dipole of the electron, and second from the interaction of the electron’s charge with the intrinsic magnetic dipole of the proton. Here we will simply give a rough estimate of the size of the classical field angular momentum of this system, and compare its size to $\hbar/2$. This is somewhat contrary to the previous section where it was shown that such semiclassical arguments are not to be trusted. However, given the more complex nature of the systems now considered, a rigorous treatment would require a much more intensive analysis. Taking the electron’s dipole moment to be along the $z$ axis, Eq. (4) gives the magnitude of the field angular momentum along the $z$ axis as

$$\mathbf{L}_{\text{em}} = (eM/c)(1 - \cos^2 \theta) \mathbf{\hat{z}}.$$ 

The maximum for this magnitude occurs when $\theta = \pi/2$ with $\mathbf{L}_{\text{em}} = (eM/cR)$. Now $M = (e\gamma^2/4m_e c)$ and we will take $R = 8.0 \times 10^{-9}$ cm, which is the expectation value for $R$ for the hydrogen atom ground state. We take the gyromagnetic ratio for the electron as $g = 2$. Inserting numbers into this (we use the units of Ref. 2 where $e = 4.8 \times 10^{-10}$ esu and centimeters and grams are to be used) gives a value of $\mathbf{L}_{\text{em}} \approx 3.5 \times 10^{-3} (\hbar/2)$, which is five orders of magnitude less than the smallest possible unit of angular momentum of $\hbar/2$. The contribution to the field angular momentum coming from the interaction of the electron’s charge with the proton’s magnetic moment will be smaller than this by three orders of magnitude since the mass of the proton is approximately 2000 times that of the electron (the gyromagnetic ratio for the proton is larger than for the electron, but they are still the same order of magnitude). In addition the field angular momentum coming from the interaction of the electron’s charge with the proton’s dipole moment subtracts from the field angular momentum coming from the interaction of the proton’s charge with the electron’s dipole moment. Given the small size of field angular momentum, from the above order of magnitude calculation, compared to $\hbar/2$, one could argue that the electromagnetic field angular momentum does not play a role in the hydrogen atom or in any atomic size system in general.

Since the field angular momentum has a $1/r$ dependence, one expects that as the system gets smaller this field angular momentum should become progressively more important. One system where the dipole field angular momentum may play a role is in quark bound states such as the proton or neutron. There are many factors that in our simple analysis would tend to change the magnitude of the field angular momentum of the quark bound states as compared to the atomic bound states. We will make of list of these factors, and discuss each in turn.

(a) Nuclear size—Since the nuclear size scale is five orders of magnitude smaller than the atomic size scale (i.e. $R_{\text{nuclear}} \approx 10^{-13}$ cm compared to $R_{\text{atomic}} \approx 10^{-9}$ cm) this would tend to increase the field angular momentum by a factor of $10^5$, making it on the order of $\hbar/2$.

(b) Quark magnetic dipole moments—The quark dipole moments can be estimated by fitting experimentally measured baryon dipole moments with the up, down, and strange quark dipole moments. In this way one obtains $M_p \approx 1.85 M_N$ for the up quark moment and $M_d = -0.97 M_N$ for the down quark moment. $M_N = e\hbar/2m_{\text{proton}} c$ is the nucleon magneton. Since this involves the mass of the proton, this tends to reduce the field angular momentum by a factor of $10^7$.

(c) Number of constituents—Compared to the hydrogen atom system which had two contributions to the field angular momentum (proton’s charge–electron’s dipole moment, and electron’s charge–proton’s dipole moment) the nucleon bound state will have six contributions to the field angular momentum (the charge of each quark with the dipole moment of each of the other two quarks). Unlike the hydrogen atom system, where the contribution to the field angular momentum from the electron’s charge interacting with the proton’s dipole moment made essentially no contribution, in the nucleon system all the contributions would be important since the up and down quarks have should have magnetic moments that have a similar magnitude. Whether the increased number of constituents would increase or decrease the field angular momentum would depend on whether the quark spins (and therefore their magnetic moments) were arranged so that the field angular momenta of each quark–quark subset added to or subtracted from the field angular momenta of the other quark–quark subsets.

(d) Quark charges—The quark charges are either two-thirds or one-third of the electron’s charge. This would tend to slightly decrease the field angular momentum as compared to the hydrogen atom system. These rough estimates indicate that, relative to atomic scale systems, the decreased size of the nuclear system increases the field angular momentum by five orders of magnitude. However, the estimates of the quarks’ dipole moments in point (b) would decrease the field angular momentum by two or three orders of magnitude. Thus, although for nuclear scale systems the field angular momentum should be larger than for atomic scale systems, it is still somewhat unlikely that it would play a significant role even in nuclear scale systems. Nevertheless, one might postulate that this field angular momentum may be a possible explanation for the nucleon spin crisis. In 1987, experiments at CERN (see Ref. 12 for a review) indicated that a considerable fraction of the proton’s or neutron’s spin does not come from the spins of the up and down valence quarks. This is contrary to the naive picture where the nucleon’s spin of $1/2$ comes from two of the valence quarks having spins of $+1/2$ while the other quark has a spin of $-1/2$. Possible explanations for this involve placing the extra internal angular momentum with the nonvalence quark–antiquark pairs, or with the gluon spin/orbital angular momentum. The alternative put forth in this paper (that the extra angular momentum may reside in the electromagnetic field angular momentum) is equivalent to saying that the extra internal angular
momentum of the nucleon comes from photons rather than gluons. Actually the present argument could more realistically be applied to the gluons as well. Just as the quarks carry ordinary electric charges they also carry color charges. If one took an approximation where radiative corrections were essentially ignored, then one would get a strong force version of the Breit–Fermi potential.\(^3\)\(^4\) This would give an interaction between the color charge and color magnetic dipole in analogy with the electromagnetic interaction, which would lead to a field angular momentum coming from the color charges and dipoles. In Ref. 13 this procedure was used to explain the mass splittings between hadrons in terms of the color hyperfine interaction between quarks. Although this approach lacks a firm theoretical foundation (i.e., why is one justifying in making a one-gluon exchange approximation to get this QCD Breit–Fermi potential\(^10\))?, it nevertheless gives results that are qualitatively and also quantitatively in agreement with the experimental hadron masses. Using the same approximation in the present case would indicate that color charges and color dipoles should give rise to a field angular momentum as in the electromagnetic case. If the color charges interacting with the color magnetic dipole moments did result in a field angular momentum, then this internal angular momentum would certainly play a role in the net spin of the nucleon, since the color charges are larger than their electromagnetic counterpart (i.e., the strong fine structure constant, \(\alpha_s\), is larger than the electromagnetic fine structure constant, \(\alpha_{em}\)). Assuming that this field angular momentum is the source of the nucleon spin puzzle, one way to distinguish between whether the electromagnetic or strong interaction is the source of the extra angular momentum is to compare the proton and neutron spin structure. If the electromagnetic field is the source then one would expect a difference between the proton and neutron since they have different charge distributions, while if the color field is the source then the proton and neutron should be roughly similar with respect to this field angular momentum.

\section*{V. CONCLUSIONS}
The semiclassical method\(^3\) of restricting the field angular momentum of a magnetic charge—electric charge system to some integer multiple of \(\hbar/2\) is often used in textbooks\(^6\)\(^7\) and articles to arrive at the Dirac quantization condition. By applying the same argument to the more realistic system of an electric charge and a magnetic dipole we run into the problem that there are an infinite number of incompatible quantization conditions for different positions that the dipole and charge can take up relative to one another. This comes about since the field angular momentum for the dipole–charge depends on the distance, \(R\), and on the relative orientation, \(\theta\), between the charge and dipole. This indicates that it may be somewhat of an accident that this semiclassical method works for the charge–monopole system, and in any case shows that caution should be used when applying this method to other systems. It is shown that even though the dipole–charge system has trouble with the semiclassical quantization method, the charge plus field angular momentum of the system (as well as the total system angular momentum) does obey the standard commutation rule, \([L_i, L_j] = i\hbar\epsilon_{ijk}L_k\), so that it is a proper quantum mechanical angular momentum. Finally some physical systems are discussed where this dipole field angular momentum could play a role. From rough estimates it is argued that for atomic size systems, such as the hydrogen atom, one would not expect this field angular momentum to be significant, while for nuclear systems, such as the proton, it could play a role. This may provide a possible explanation for the nucleon spin puzzle, especially if one considers a QCD version of the field angular momentum.

\section*{ACKNOWLEDGMENTS}
I would like to thank Chris Ashman and Herbert Schmidt for useful suggestions and comments.

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\section*{TEACHERS’ MISTAKES}
Nothing is more bracing for students than to discover the fallibility of their exalted teachers. Students, God knows, are brimming with their own human weaknesses, and if their great mentors can make mistakes, well then, anything might happen.